Exeter Math Institute

Hands-on Algebra
Part A

Hands-on Algebra I – Part A

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Guess and Check Part A

Solving word problems often takes more time and effort that a computational math problem. However, they often can be done by thinking about how you might check an answer if you finally found one. In solving the following problems, try to suspend any previous methods that you might have learned and follow the directions step-by-step.

1. The length of a certain rectangle exceeds its width by exactly 8 cm. The perimeter of the rectangle is 66 cm. What are its dimensions?

   Although you may be able to solve this problem using a method of your own, try the following approach, which begins by guessing the width of the rectangle. Study the first row of the table below, which begins with a guess of 10-cm. for the width. Now make your own guess for the width and use it to fill in the next row of the table. Be sure to show the arithmetic in detail that you used to complete each entry. If this second guess was not correct, try again.

width	length	perimeter	desired perimeter	check?
10 cm.	10 + 8 = 18 cm.	2(10)+2(18) = 56	66	no

Even if you have guessed the answer, substitute a *w* in the first column and fill in the length and perimeter entries in terms of *w*.

Finally, set your expression for the perimeter equal to the desired perimeter. Solve this resulting equation and thus solve the given problem.

This approach to creating equations and solving problems is called the ***guess-and-check*** method.

2. A portable CD player is on sale for 25% off its original price. The sale price is $30. What was the original price?

In the table below, one guess has been made, which was not the correct answer. Make a guess of your own and put it in column 1. Continue to fill in across the row with the appropriate values. If your guess does not yield the correct solution, use another row of the table and guess again. Be sure to show your calculations in the table.

Finally, place a variable in the first column and continue across the row, performing the same operations with the variable as you did with numbers, as far as possible.

Form an equation by setting your expression in the "sale price" column equal to the number in the "given sale price" column. Solve the resulting equation in the space below the table and thus solve the given problem.

Original price(**guess**)	25% of original price	sale price	given sale price	equal?
60	.25(60)=15	60-15=45	30	No

3. A group of students are in a room. After 12 students leave, 3/5 of the original group remain. How many were in the room originally?

Once again, begin by making a guess at the correct answer. Eventually, put a variable in the "guess" column, complete each column as in the previous examples and finally form an equation that can help you solve the problem. Remember to show your work, both in the table and in solving the equation.

Original number of students(**guess**)	number who left	number remaining	3/5 of the original	check?

4. There are 1800 students in a high school. The ratio of male to female students is 4:5. How many boys are there?

As in previous examples, begin by making some guesses and then use a variable to form an equation. Solve the equation in the space below the chart.

number of boys(**guess**)	total number in school	number of girls	ratio of boys to girls	is ratio 4/5?

5. At noon, you start out walking to a friend's house at 4 mph. At the same time, your friend starts biking towards you at 12 mph. If your friend's house is 8 miles away, how much time will elapse before you two meet?

In the table below, put your own headings on the columns before making a guess. It is usually a good idea to label the first column with what you are trying to find. After putting the titles on the columns, make your guess, form your equation and solve it as in the previous problems. You may use fewer or more columns than are shown, if you wish.

6. Kim just bought ten gallons of gasoline, the amount of fuel used for the last 360 miles of driving. Being a curious sort, Kim wondered how much fuel had been used in city driving (which takes one gallon for every 25 miles) and how much had been used in freeway driving (which takes one gallon for every 40 miles). Determine the number of gallons used on each part of the trip.

   This time, try to determine what headings you might put at the top of each column. Start by thinking about what you wish to find and use that as the title of the first column. Thinking about what else you can determine should help you with the headings of the other columns. There are several possibilities.

   Finally, form and equation and solve it.

7. A train is leaving in 12 minutes and you are 1 mile from the station. Assuming you can walk at 4 miles per hour and run at 8 miles per hour, how much time can you afford to walk before you must begin to run in order to catch the train? Use the *guess-and-check* method to solve this problem.

What is a Function?

Teacher Introduction:
Mars Candy Company manufactures M&M candies. These are small chocolate candied coated in a colored hard shell with the letter "M" imprinted upon them.
https://www.youtube.com/watch?v=xVJTIz7Z3Ds
The video does not show the letter "M" being stamped onto each candy. To do this, each blank M&M enters a machine while on a conveyor belt. The M&Ms are secured in an individual "dimple" or seat. A rubber roller prints the letter "M" on each M&M as the conveyor belt moves under the roller. Each printer can stamp 2.5 million M&Ms per hour.
What does this have to do with functions? Each blank M&M is an input. It is inputted into the printer machine. The printer changes the input by stamping an "M" onto it. The candy with the "M" stamped on it is then the output of the machine. In a sense, the printer machine could also be called a function. It accepts inputs, acts upon them, and then puts out the output.

But are all input/output relationships functions? Let's explore this.
A function is a relationship – the input goes into the function and it creates a predictable output. For example, each student in the class should stand in a line. They are members of the domain, or the input. Place 12 post-it notes on the wall, each labeled with a month of the year. Ask students to stand in front of the post-it note that has their birthday month written on it. Students will distribute themselves. Ask students, "Did anyone not have a place to go? Did anyone need to choose between two or more places to go?" Emphasize that each input had exactly one output. Ask, "Does each month correspond to just one student? Are there any months with no students? Are there any months with many students?" The function is the relationship that has only one output for each input. The function in this case is the relationship between student and birthday month. Each student has a unique birthday month.

Now put post-it notes on the wall that represent colors, e.g. white, black, blue, grey, red, orange, green, pink, purple, brown, etc. Ask students to stand next to the post-it note that has the color of their shirt or top. This should create some confusion. Many students will not know where to go because their shirts are more than one color. There is still a relationship between students and the color of their shirt, but it is not unique. Many colors are related to each student. In this case, the relationship is not a function.
In this case, the domain of the function is the group of shirts worn by students. The range is all possible colors of the shirts worn by students. Now let's restrict the domain to only the shirts that are a single color. Ask those students to stand by the post-it note that is labeled with the color of their shirt. Ask, "Does each student know where to stand? Is there more than one place each student can stand?" Now that we have restricted the domain, that is, the possible values of the input, this relationship is a function because there is only one possible place for each student to stand.

Student Instructions:

1. Find a partner. Develop two ideas of relationships. Describe the input and the output for these relationships. Decide with your partner whether these relationships are functions.

Input	Output	Is this a function?
Example: Number of dogs	Number of dog ears	Yes, because the number of dog ears can be exactly determined by the number of dogs
Example: Name of mother	Name of child	No, because a mother could have more than 1 child
3.		
4.		

2. Now let's explore some mathematical relationships. Suppose you plant a tree outside. A local biologist told you that your tree will grow 20 cm each year. Let's let "*h*" be the label for the height and "*a*" be the label for the age of the tree in years.

   Describe in words the relationship between age, in years, and height, in cm:

3. What is the input (independent) variable?

4. What is the output (dependent) variable?

5. When the tree is one year-old, how tall do you expect it to be?

6. When the tree is two years old, how tall do you expect it to be?

7. When the tree is three years old, how tall do you expect it to be?

8. When the tree is ten years old, how tall do you expect it to be?

9. When the tree is 17 years old, how tall do you expect it to be?

10. In terms of *h* and *a*, write an equation that describes this relationship mathematically. That is, if someone knew the age of the tree, how could they calculate the height using your equation?

11. What are some reasonable ages for the tree to be? How young can it be? How old can it be? The span of these ages is called the domain. Write a plausible domain of ages as an inequality.

12. What are some reasonable heights for the tree to be? How short can it be? How tall can it be? The span of these heights is called the range. Write a plausible range of heights as an inequality. Does the equation you wrote in question 10 provide plausible data for the growth of a tree? Why or why not?

13. Let's look at a new relationship. Suppose your class wants to fundraise for a school dance, so you decide to design and sell t-shirts. It costs $60 to secure the vendor to produce the t-shirts and then you can make a profit of $3 on each t-shirt you sell.
    What are the two variables involved in this problem?

    Create a label or name for each variable.

14. Write an equation that describes the relationship between the number of t-shirts sold and the amount of money you make or lose.

15. Fill in the chart for some possible values and plot the relationship:

Input	**Output**

16. Is this relationship a function? Explain why or why not.

17. What are the axes intercepts? What is the meaning of the intercepts in this context?

Domain and Range

You will be given one 6-sided die and cards with function rules on them.

1. If you roll your die once, what are the possible values that are face up? List these values:

2. These values are the input values. The list above is the domain of the function, that is, the list of all of the possible input values.

3. Now roll your die again and record the number that appears face up:

   Draw a function rule from cards that are provided. Apply the function to the input value of your roll. What is the output value? Record your results in the table below.

4. Roll the die again and record the input value:

5. Apply the same function rule to the input value. What is the output? Record your results in the table below.

   Repeat this process 3 more times and record the output each time.

	Input	**Output**
Trial 1		
Trial 2		
Trial 3		
Trial 4		
Trial 5		

6. Record all possible output values that you could obtain using this function rule:

The values above represent the range, that is, the set of all possible outputs from a function rule.

7. Draw a new function rule card. Roll the die and apply your rule. Repeat this process 5 times and record the output each time.

	Input	**Output**
Trial 1		
Trial 2		
Trial 3		
Trial 4		
Trial 5		

8. Record the range of this function rule:

9. You and a partner are now given a bouncy ball and a meter stick. By placing the end of the meter stick on the floor, hold the bottom of the bouncy ball a certain height from the floor. Release the ball and record how high the bottom of the ball bounces to the nearest mm. Fill in the table below:

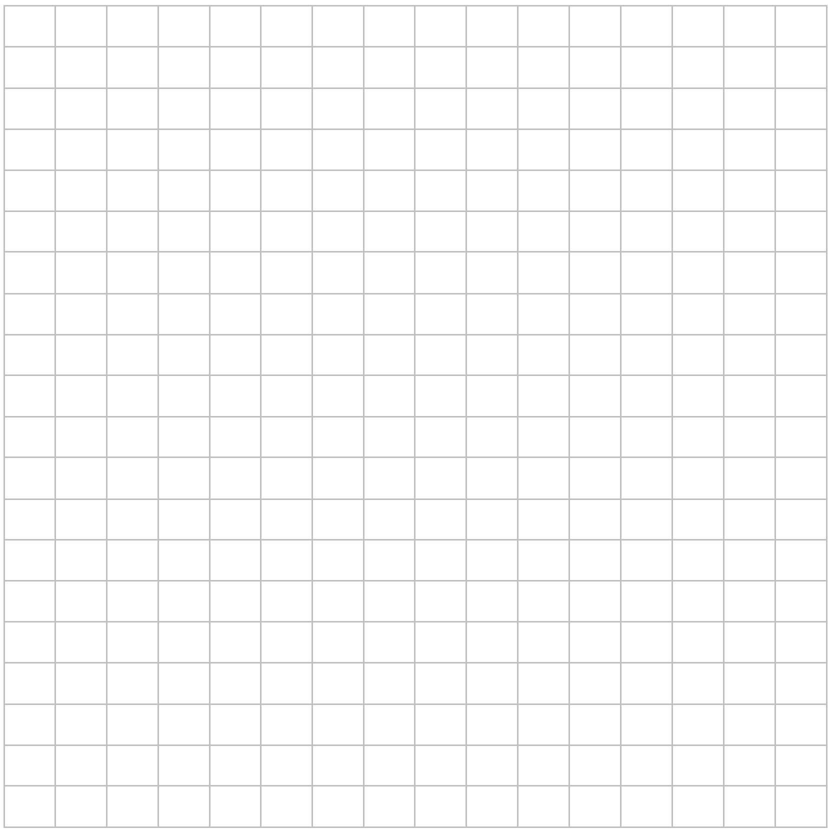
Height of bouncy ball from floor	**Height of bounce**

10. Is the relationship between the height of release and the height of the bounce a function? Why or why not? Explain.

11. What is the domain?

12. What is the range?

13. Plot the height of the bounce vs. the height of the release:



14. Using a ruler, draw a line that you believe best fits the points on your plot above.

15. Is this function increasing or decreasing? Why?

16. Write an equation for the line you have drawn.

17. What is the slope of the line?

18. Interpret the meaning of the slope in the context of the height of the bounce and the height of the release. That is, describe what the slope means in terms of these two heights.

19. What are the intercepts?

20. Describe what the intercepts mean in the context of the height of the bounce and the height of the release. Do the intercepts make sense for this situation?

Geoboard Slope Activity I

Materials **needed: each pair or person needs one 11-pin geoboard or arrange four 7-pin geoboards so that they represent the four quadrants of the *x*-*y* coordinate plane.**

Directions:

Put rubber bands vertically and horizontally through the center pin to represent the *x* and *y*-axis. The individual pegs of the geoboards represent points with integer coordinates. These points are known as *lattice points*, or *grid points*. In this activity, the student will take the end points of the rubber bands and place them on the correct pegs so that a line segment is represented by the rubber band.

1. **Some general questions.**

a) Create segments with **different** positive slopes, one in each quadrant, by placing end points of rubber bands in appropriate locations. Record the coordinates of the end points of the rubber bands A and B and the slope of $\overline{AB}$ in the table shown. Write a description of segments that have positive slope in the space below.

Quadrant	I	II	III	IV
Point A				
Point B				
Slope of $\overline{AB}$				

____

____

b) Create segments with **different** negative slopes, one in each quadrant, by placing end points of rubber bands in appropriate locations. Record the coordinates of the end points of the rubber bands A and B and the slope of $\overline{AB}$ in the table shown. Write a description of segments that have negative slope in the space below.

Quadrant	I	II	III	IV
Point A				
Point B				
Slope of $\overline{AB}$				

____

____

c) Write the coordinates for *A* and *B* for three different examples of line segments with zero slope.

____________________ ____________________ ____________________

How would you describe lines that have zero slope?

d) Write the coordinates for *A* and *B* for three different examples of line segments with undefined slope.

____________________ ____________________ ____________________

How would you describe lines that have undefined slope?

2. Remove all the rubber bands other than the *x* and *y* axis from the previous section of this activity. Put one end of a rubber band on the indicated point *A*, and the other end on a point *B* somewhere on the board so that the slope of the line *AB* is the number in the left column of the table below. Note that there may be more than one solution. Record as many as you are able to in the indicated space (next page).

	Slope	**Coordinates of *A***	**Coordinates of *B***
(a)	$\frac{2}{1}$	(0,0)	
(b)	$\frac{1}{2}$	(–4,3)	
(c)	$-\frac{3}{1}$	(–1,1)	
(d)	$\frac{2}{3}$	(3,2)	
(e)	Undefined	(3,1)	
(f)	$-\frac{3}{4}$	(–1,1)	
(g)	$-\frac{3}{2}$	(-3,2)	
(h)	$\frac{5}{3}$	(4,1)	
(i)	0	$(-3,1)$	
(j)	1	$(4,-2)$	

Geoboard Activity for $y = mx + b$

Materials needed: **each pair or person needs one 11-pin geoboard or arrange four 7-pin geoboards so that they represent the four quadrants of the *x*-*y* coordinate plane.**

Directions:

Put rubber bands vertically and horizontally through the center pin to represent the *x-axis* and *y-axis*. The individual pegs of the geoboards represent points with integer coordinates. These points are known as *lattice points*, or *grid points*. In this activity, the student will place the endpoints of the rubber band on grid points *A* and *B* so that the rubber band will be on the line whose equation is given in slope-intercept form in the left column of each table.

I. In this section, one of the endpoints of the rubber band is always the origin (0,0). Determine the coordinates of point *B* for the other endpoint of the rubber band, and record its coordinates in the table below. Note that there may be more than one solution. Record as many as you are able to in the indicated space.

	Equation of Line	Coordinates of A	Coordinates of B
(a)	$y = \frac{2}{3}x$	(0,0)	
(b)	$y = -3x$	(0,0)	
(c)	$y = \frac{3}{2}x$	(0,0)	
(d)	$y = 0$	(0,0)	
(e)	$y = -\frac{5}{4}x$	(0,0)	
(f)	$x = 0$	(0,0)	
(g)	$y = \frac{4}{3}x$	(0,0)	
(h)	$y = -\frac{2}{5}x$	(0,0)	
(i)	$y = 4x$	(0,0)	

II. In this section, not only do you have to determine both endpoints *A* and *B* for the rubber band to be on the given line, but none of these lines contain the origin. Determine the coordinates of both *A* and *B*, and record their coordinates in the table below.

	Equation of Line	Coordinates of A	Coordinates of B
(a)	$y = \frac{4}{3}x - 5$		
(b)	$y = -1.5x + 4$		
(c)	$y = 2$		
(d)	$y = \frac{7}{4}x - 3$		
(e)	$x = 4$		
(f)	$y = -\frac{4}{5}x + 1$		
(g)	$y = 2x - 3$		

III. Now the *y*-intercept is not the origin as in part I, and it is not even an integer as in part II. Yet, you can find integer coordinates of A and B.

	Equation of Line	Coordinates of A	Coordinates of B
(a)	$y = \frac{3}{2}x - \frac{5}{2}$		
(b)	$y = -0.6x + 1.8$		
(c)	$y = \frac{1}{4}x - \frac{3}{4}$		
(d)	$y = \frac{4}{3}x + \frac{5}{3}$		
(e)	$y = 0.375x - 0.5$		

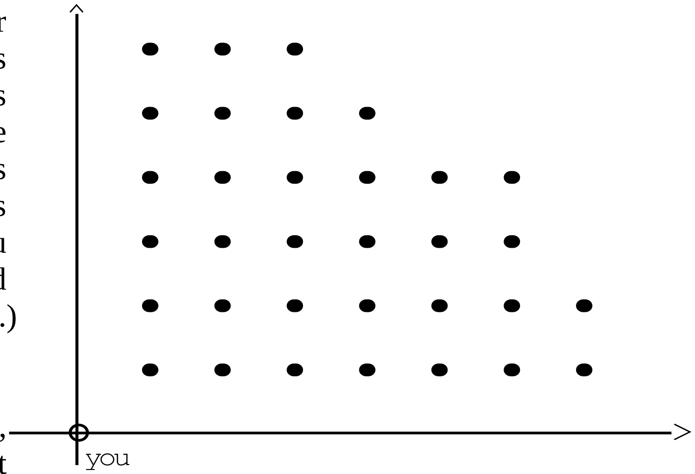
IV. If your geoboard was infinite in its dimensions, would the line $y = 1.2x + 1.5$ contain any lattice points? State the coordinates of such a point, or explain why none exists.

The Orchard

Materials needed: A sheet of graph paper and a ruler for reach participant.

Directions:

An orchard is planted in a rectangular grid such as seen at the right. The trees are all evenly spaced along the rows and columns. You, the observer, are sitting in a swivel chair at point O. As you swing around in your chair, is there any line of sight along which you would see no trees? (Assume the grid continues infinitely to the right and up.)



To help us approach this problem, consider the grid to be the first quadrant of a coordinate axis system. You, the observer, are at (0, 0) and the trees are located by coordinates that are non-zero integers. Thus the trees in the first row are at (1, 1), (2, 1), (3, 1) . ..... , the second row at (1, 2), (2, 2) . . .... and so forth. It may be helpful to use the edge of a ruler to represent a line of sight.

1. List the coordinates of four trees that you can see.

________ ________ ________ ________

2. List the coordinates of four trees that you can not see because they are hidden by other trees.

________ ________ ________ ________

3. Can you see a tree at (5, 3) ? _______ (6, 3) ? _______ (6, 8) ? _______
(2, 10) ? _______ (2, 15) ? _______ (21, 24) ?______

4. You can see a tree at (1, 2). List some trees hidden by the tree at (1, 2) .

____

5. What characterizes the coordinates of trees hidden by a given tree?

____

6. What characterizes the coordinates of trees that you can see?

____

7. If you look along the line of sight determined by each of the following equations will you see a tree? If so, list the coordinates of the first tree seen. If not, explain why no tree is seen.

a) $y = 2.3x$

b) $y = 4\frac{2}{3}x$

c) $y = 0.\overline{3}x$

d) $y = 0.002x$

8. Return to the original question. Could you write the equation of any line of sight along which no trees would be seen? Explain.

____

____

Was Leonardo correct?

Leonardo da Vinci (born 1452) wrote instructions to artists about how to proportion the human body in painting and sculpture. Three of Leonardo's rules were:

- Height equals the span of the outstretched arms
- Kneeling height is three-fourths of the standing height
- The length of the hand is one-ninth the height

Discuss this as a group. Do these proportions seem reasonable? Is Leonardo really suggesting that there is a linear relationship between these lengths?

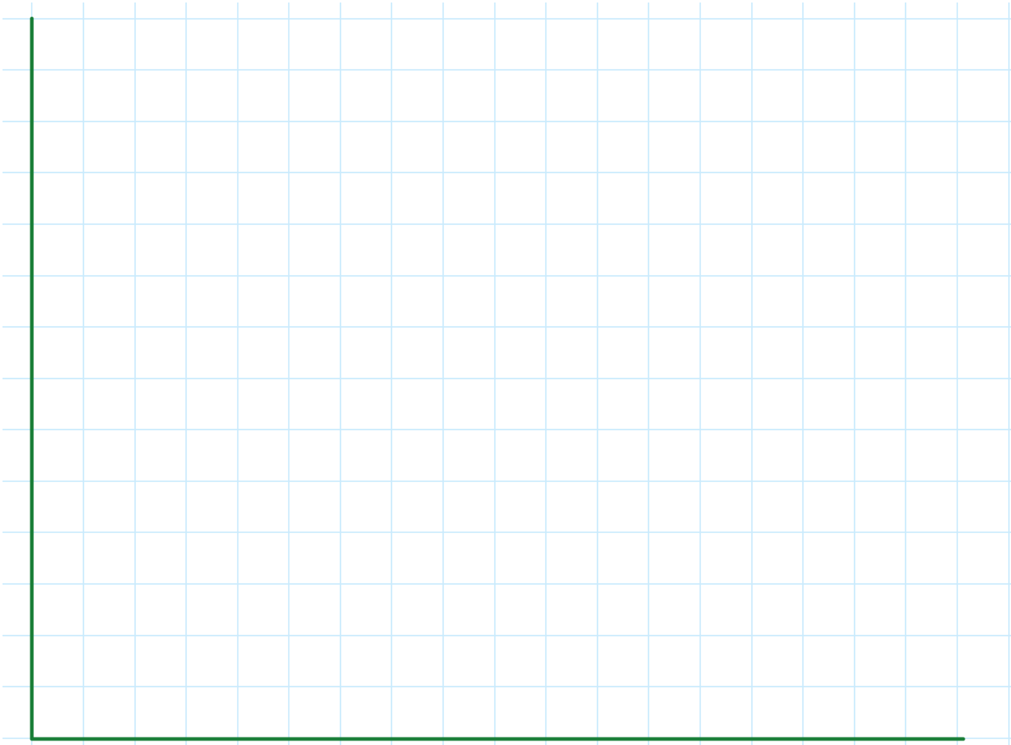
1. Decide as a class which units of measure you will use. Then working with a partner, measure your height, kneeling height, arm span and length of hand and record it in the table below:

My Measurements

Height	**Kneeling height**	**Arm span**	**Hand length**

2. On the board, make a data table that includes measurements from everyone in the class.

3. Make three scatterplots of the data: arm span vs. height, kneeling height vs. height, hand length vs. height.



4. Describe the relationship you see in each of the three graphs. Discuss this with a partner. Do you agree? Write down your observations.

5. With a ruler, draw a line on your graph that you believe best "fits" or "models" the data.

6. Write an equation for each of your three lines:
    a. Line for arm span vs. height:

    b. Line for kneeling height vs. height:

    c. Line for hand length vs. height:

7. What is the rate of change of each of your three lines? What rate of change did Leonardo expect you to find? Does your rate of change agree with Leonardo's expectations?

8. Interpret your three rates of change. What do they mean?

9. What are the three intercepts? What would you expect them to be? Are they the same as your expectations? Why or why not?

Springs

Materials needed: For each pair or group, one spring(½ of a Slinky works), a bucket, 2 meter sticks, M&Ms or substitute to fill the buckets.

Directions:

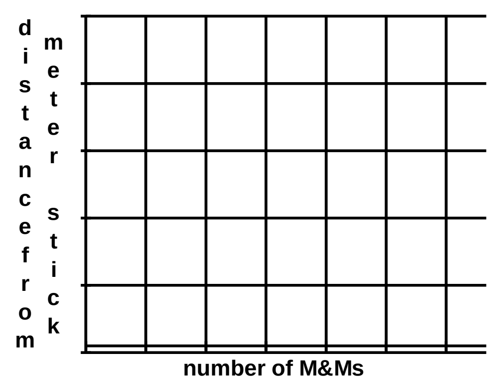
1. Place one meter stick between two chairs and slip one end of the slinky over it so that it hangs down. Hang the bucket from the other end of the slinky.

2. Measure the distance, in centimeters**, from the bottom of the empty cup up to the meter stick** that is holding the slinky. <u>*Record your answer*</u> here ____________.

3. For trial number 1, place 4 M&Ms in the container and measure the distance from the meter stick to the bottom of the bucket. Record your results in the table at the right. Continue to place more M&M candies in the bucket, four at a time. Each time, measure and record the distance from the meter stick to the bottom of the canister.

trial	Total # of M&Ms	Distance from meter stick	
1			
2			
3			
4			
5			
6			

4. Using your calculator, make a scatterplot of the data found in step 3. In your graph, plot the total number of M&Ms on the horizontal axis and the distance from the meter stick plotted on the vertical axis. Make a copy of your graph on the graph provided on this paper. Label the points with their coordinates.



5. Your data should look like it falls on a straight line. Select the two points from your data that seem to most represent the data and write their coordinates here.

   (first point) ____________ (second point) _______________.

   Find the equation of the line through these two points, showing your work here.

   Write your final equation here: y = __________________________

6. Graph your line along with your scatterplot. Does your line approximate your data?

7. The slope of your line is a “rate”. Explain the meaning of the specific number you computed for your slope in terms of M&Ms and the distances measured.

8. What is the significance of the y-intercept of your line? How is it related to the measurement made in step 2 of this lab?

9. Using your equation, predict the distance that the bucket would be from the meter stick if 36 M&Ms were placed in the bucket.

10. Using your equation, find **the least number** of M&Ms that would be needed to stretch the bucket and spring to a total length of at least 100 cm.

11. Fill in the fourth column of data by subtracting your initial length of the spring and bucket without any candy, from each length in your original data. Plot this new data on the vertical axis while leaving the horizontal axis as the total number of M&Ms. Pick the two points whose x-values are the same as the points chosen in question 5 and again form an equation of a line that seems to fit the data the best.

    Write the equation of the line ______________________________

12. Graph your equation. What should the y-intercept be? ___________________

13. Compare the graph of this line with the line graphed in question 6. How are they alike and why is this so?

14. Explain the significance of the slope in this equation relative to the spring and the M&Ms.

Rings

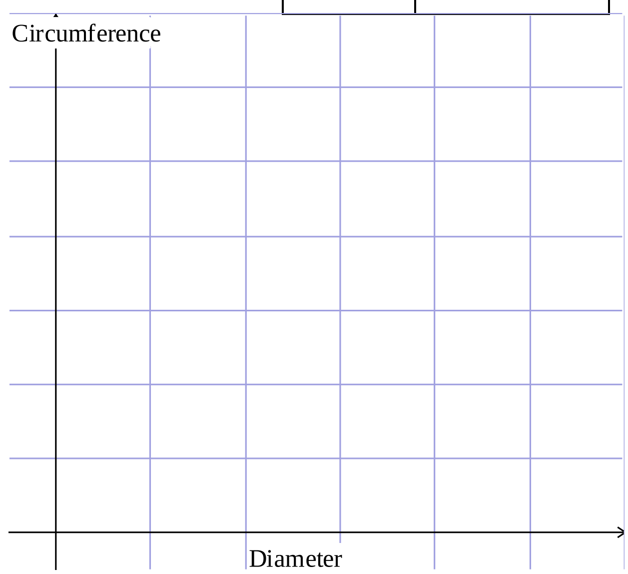
Materials needed: For each group, set of 5 rings of varying diameters (plastic lids, covers etc.), string to measure circumferences (alternately, you may roll the disks along the meter stick to measure the circumference), meter stick.

Directions:

1. For each of five different size lids, measure the diameter of the lid in centimeters to the nearest tenth. Then measure the circumference of the lid also to the nearest tenth of a centimeter. Record your data in the table at the right.

diameter	circumference

2. Plot your data on the grid at the right. Use the diameters as the independent variable.



3. Now graph the data using your calculator. Is your calculator graph similar to your graph in Step 2? What pattern does the data seem to follow?

____________________________ _____________________.

4. Select two points from your data that seem to be representative of the pattern and record them here.

first point _____________ and second point ________________.

5. Find the equation of the line through these two points, showing your work here.

Write your equation in slope-intercept form: y = ____________________________

6. Graph your equation along with your data. Explain what the number you computed for the slope means in terms of the measurements you made.___________________________________

____

Is the number close to what you might have predicted? _____ Explain. __________________

____

7. What is your y-intercept? ________ If your work was very accurate, what would you expect the y-intercept be? ________ Explain. ____

8. If you had a lid that was 250 centimeters in diameter, what would be its circumference according to your equation?

9. According to your equation, what should be the diameter of a lid that has a circumference of 400 centimeters?

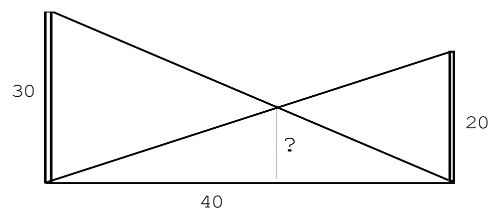
10. Bert computed the equation for his data to be $y = 3.13x$. Ernie found his equation to be $\hat{y} = 3.13x + 0.02$. Each would like to use their equation to find the circumference of a lid whose diameter was found to be 2.5 feet. Explain why Ernie must convert the 2.5 feet to cm while Bert does not.

The Telephone Poles Problem

Materials needed: a sheet of graph paper and a straightedge for each participant.

Read the following problem

1. Two telephone poles are erected perpendicular to the ground and 40 meters apart as shown at the right. The poles are 30 meters and 20 meters tall respectively. Two of the supporting wires are shown, each running from the top of one post to the bottom of the other.



The goal of this exercise is to find how high the crossing point of the two wires is off the ground.

a. Begin by drawing the picture above on graph paper. Put the ground on the x-axis and the 30-meter pole along the y-axis. Draw the picture to an appropriate scale so that it fits conveniently on the paper.

b. Find the equation of the supporting wire for the 20-meter pole. ________________________

c. Find the equation of the supporting wire for the 30-meter pole. ________________________

d. Graph the two equations you just formed on your calculator. Choose a window that allows you to duplicate the picture above.

e. Using your calculator, find the answer to part (a) and record it here. ____________________

f. Verify your calculator answer by solving the two equations simultaneously in the space below.

(b)The public service commission has stipulated that the meeting point of the wires must be 15 meters off the ground. With the given heights of the poles, would it be possible to position the poles either closer together or further apart so that this requirement could be met? Explain your answer.

Here are some questions to aid your thinking.

7. Do you think the poles should move closer together or farther apart to bring the intersection point of the wires higher? ____________ Begin by trying a new distance between the poles and making the appropriate changes in your equations. Graph the new equations and find their intersection.

   First try:

   New distance between poles _________

   New equation of the supporting wire for the 20-meter pole. _______________________

   New equation of the supporting wire for the 30-meter pole. _______________________

   Height of intersection off the ground _______________

   Second try:

   New distance between poles _________

   New equation of the supporting wire for the 20-meter pole. _______________________

   New equation of the supporting wire for the 30-meter pole. _______________________

   Height of intersection off the ground _______________

   What is your conclusion? (If you are not sure, try a third example.) Write your conclusion:

   ____

8. To verify your conclusion, let D be the distance between the poles. Write an expression in terms of D for the slope of the wire to the top of the 20-meter pole and then do the same for the slope of the wire to the top of the 30-meter pole.

   Slope of wire to the 20-meter pole _______ slope of wire to the 30-meter pole ___________

   Now write the equation of the wire for the 20-meter pole in terms of D. _______________

   Write the equation of the wire for the 30-meter pole in terms of D ____________________

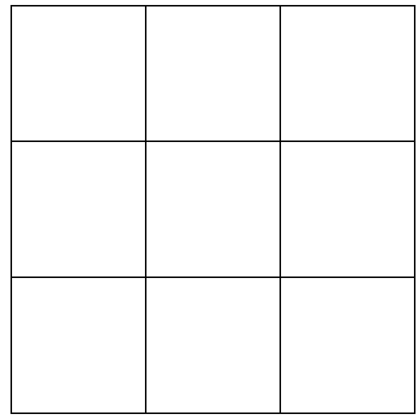
   Verify your conclusion by setting the two equations equal and solving for x in terms of D. Then substitute your value for x into one of the original equations. Explain your result.

Rates of Change - Part 1

Materials needed: 25 wooden blocks per group or alternately, 25 Starburst candies.

Directions:

1. Take 9 blocks and form a 3 by 3 square as shown at the right.



2. How many blocks lie on the perimeter of the figure? Record that number in the table below the figure.

3. Take 16 blocks and form a 4 by 4 square. How many blocks lie on its perimeter? Record that number in the table.

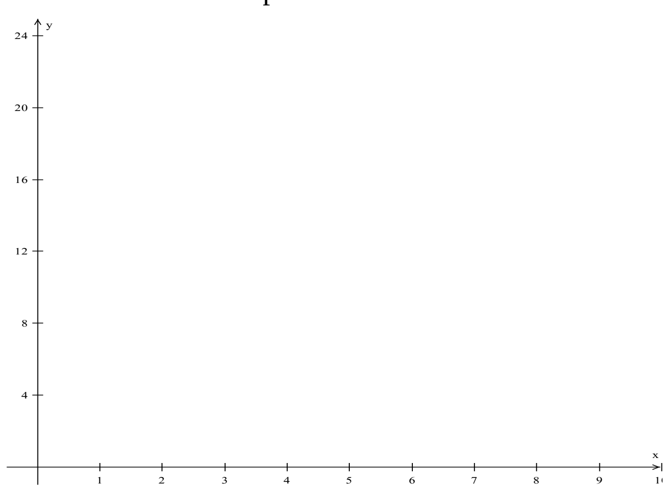
4. Take 25 blocks and form a 5 by 5 square. Again, determine the number and record it in the table.

5. If you had enough squares to form a 6 by 6 square, how many squares would there be in its perimeter? [You can use your squares to help you get the answer by partially forming the square.]

6. Try to determine a pattern to your data and fill in the values for squares of side 2 and side 7.

7. Make a scatterplot of this data on your calculator. Let *x* represent the number of blocks on the side of a square and *y* represent the number of blocks on its perimeter. Copy the scatterplot onto the graph below. Be sure to label your axes with a scale and a description of the variable.

Number of blocks on the side of square	Number of blocks on its perimeter.	Difference between entry values
3		
4		
5		



8. Determine the successive differences between the values in the second column and write them in the third column to the right of the table. What do you notice about these differences?

9. Write an equation that describes the pattern you determined in step 6 in terms of x and y. Graph the equation on the scatterplot and on your graph above. Does it go through your data?

10. In terms of the number of blocks, what is the meaning of the number you computed for the slope of the function?

11. For what value of x is $y = 0$? What does that mean in terms of the context of the problem?

12. Use your equation to predict how many squares would be in the perimeter of a square that is 12 by 12. Write your work and answer here.

13. Use your equation to predict the size of a square given that there were 56 squares in the perimeter of the square and record your work and answer here.

Rates of Change – Part 2

Materials needed: 25 wooden blocks per group or alternately, 25 Starburst candies.

Directions:

1. Return to the 3 by 3 square that you formed in Part 1. If you take away the blocks that lie on the perimeter of the square, how many are left in the interior of the original square? Record that number in the table below.

2. Now take away the blocks that lie on the perimeter of the 4 by 4 square. How many blocks are left in its interior? Record that number in the table at the right.

3. Do the same for a 5 by 5 square and a 6 by 6 square, recording your answers in the appropriate places in the table.

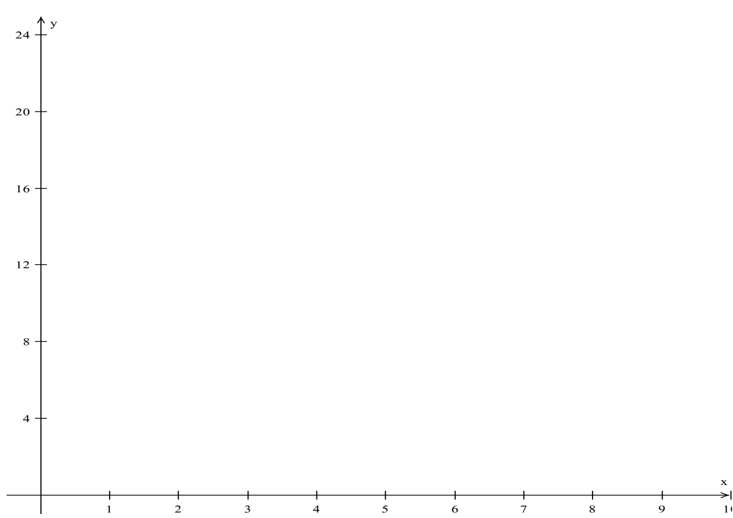
4. Try to determine a pattern for your data and fill in the values for 2 and 7 in your table using the pattern.

5. Make a scatterplot of this data on your calculator. Let *x* represent the number of blocks on the side of a square and *y* represent the number of blocks in the interier. Copy the scatterplot onto the graph below. Be sure to label your axes with a scale and a description of the variable.

Number of blocks on one side of the square	Number of blocks left
3	
4	
5	
6	

Difference between 2nd column values

Difference between 3rd column values



6. Can this data be approximated by a line very easily?

7. In the third column of the table record the successive difference between the values in the second column. Are they constant? Does the data we found form a linear function? Explain.

8. In the fourth column, record the successive differences in values of the third column. What do you notice?

9. Write an equation that describes the pattern you determined in step 4 and graph the equation along with your scatter plot. Does it go through the data? Also graph the function on your graph above.

10. The graph on your calculator is a solid curve. What is the domain of this function in the context of this problem? Does it make sense to connect the data points you plotted? Explain.

11. Use your equation to predict how many blocks there would be in the interior of a 11 by 11 square. Show your work here.

12. Use your equation to predict the original size of a square if there are 144 squares left in its interior after the squares on the perimeter have been taken away.

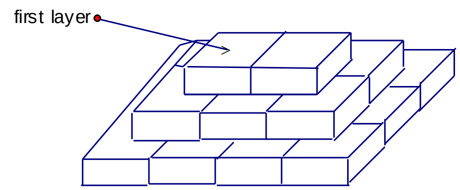
13. Take the expression you derived for the function in step 9 of Part 1 and add it to the expression you derived for the function in step 9 of Part II. Simplify your sum. Explain why the result should have been expected.

Stacking Starbursts

Materials needed: For each group, 20 starbursts or similar objects.

Directions:

1. Each group of people should have about 20 Starbursts. Each person in the group should have his/her own worksheet and should complete it as the group works through the following lab.



2. Begin by building the 3-layer figure at the right with Starbursts. Note that each layer is rectangular in shape and each Starburst covers the intersection of four Starbursts on the row below. Assume that layer #1 is the top layer of 2 Starbursts.

3. Record the number of Starbursts used in each layer in the table below.

4. You do not have enough Starbursts to actually construct a fourth layer under the three already there. However, using those in the top three layers can help you determine how many you would need for this four layer. Record that number in the table.

5. Do you see a pattern forming? Try to use this pattern to complete the table for layers 5 and 6

Layer number	Number of starbursts	Difference between 2nd column values	Difference between 3rd column values
1			
2			
3			
4			
5			
6			

6. In the third column of the table record the successive difference between the values in the second column. Are they constant? Does the data in column 3 form a linear function?

7. In the fourth column record the successive differences in the values in the third column. What do you notice? Our original data in columns 1 and 2 is therefore what degree of polynomial function?

8. Sketch a graph of your data in columns 1 and 2. It should confirm your previous answers.

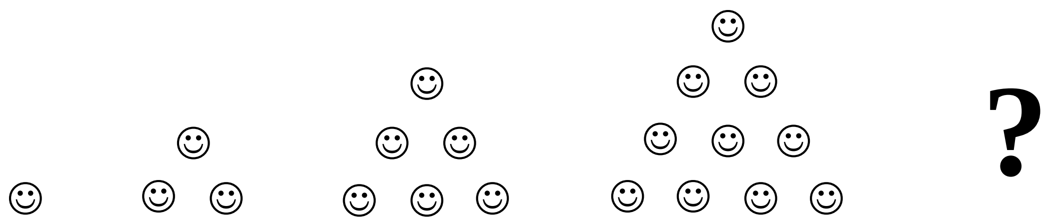
9. We would now like to write a function that would give us the number of Starbursts used, y, given the number of the layer, x. We now know it would be an equation of the form $y = ax^2 + bx + c$. To begin, using the patterns in the table, how many Starbursts would there be in the 0 layer? What then should be the value of c?

10. Using two of the sets of data points, form two equations and solve them simultaneously for a and b. You can now write the desired equation.

Smiley Faces

Materials Needed: two different colored disks or small objects for each group.

Consider triangular arrangements of smiley faces, as shown below.



(a) Continue the pattern by drawing the next triangular array.

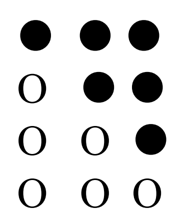
(b) Let x equal the number of smiley faces along one edge of a triangle, and let y equal the corresponding number of smiley faces in the whole triangle. Complete the second column with values that illustrate the relationship between x and y for $1 \le x \le 5$. What value of y should be associated with $x = 0$?

Number of faces on one edge x	Number of faces in whole triangle y	Number in each corres-ponding rectangle	Dimensions of each corres-ponding rectangle
0			
1			
2			
3			
4			
5			

(c) Is the relationship between x and y linear or quadratic? Explain.

(d) On the table surface, form the triangles using yellow disks, and then re-format them into right triangles by sliding the disks to the left. Then using red disks, make an inverted right triangle of the same dimensions to form a rectangle such as pictured at the right for the third entry. Do this for each triangle above.

O = yellow ● = red



(f) In column three of the table above, enter the total number of disks used in each rectangle. Be sure to include entries for 0 and 1. Be sure to label the column appropriately.

(g) In the fourth column, write the dimensions (width times height) of each rectangle.

(h) In the last row of the table, place an x in the first column. Consider the pattern in the fourth column. Write a formula in terms of x that would describe how one obtains the number in that column.

(i) Finally, can you now write an expression in the second column in terms of x that would describe how to obtain the number of smiley faces in any given triangle.

(j) The numbers 1, 3, 6, 10 … are called *triangular numbers*. Why? You have just found an equation for the triangular number relationship. Check it by replacing x with 6. Do you get the same number as there are smiley faces in the 6th triangle?

Factoring Trinomials With Tiles

Materials needed: Set of commercial tiles or hand-made tiles such as those on the next page.

Directions: Count out the following number of tiles for each group/pair of participants.

	Number needed
- small blue squares x by x called x^2-blocks	3
- large blue squares y by y called y^2-blocks	2
- small yellow 1 by 1 squares called 1's	10
- blue 1 by x rectangles called x-blocks	6
- blue 1 by y rectangles called y-blocks	5
- blue x by y rectangles called xy-blocks	3

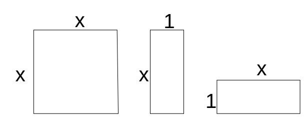
Make a rectangle

With the above resources you are going to try to make rectangles. For each set go through the following steps.

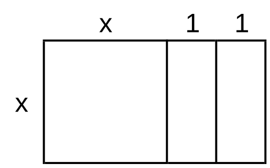
1. Write an expression for the sum of the areas of the resources. Combine like terms whenever possible.
2. Fit all the pieces together to form a rectangle.
3. Write the length and width of your new rectangle.
4. Write an expression for the area of your rectangle in the form (*length*)(*width*).
5. Write an equation that states that the sum of the areas of the individual resources (the expression you wrote in 1.) equals the area of your new rectangle (the expression you wrote in 4.)

example

Given the resources 1 x^2-block, 2 x-blocks:



1. The sum of the areas is $x^2 + x + x = x^2 + 2x$
2. Fit the pieces to form a rectangle (see lower diagram)
3. The length is $x + 2$, the width is x
4. An expression for the area is $x(x + 2)$
5. $x^2 + 2x = x(x + 2)$



you try these

1. 1 x^2-block, 3 x-blocks, 2 1's
2. 1 x^2-block, 2 x-blocks, 1 1 (yellow square)
3. 2 x^2-blocks, 4 x-blocks
4. 1 y^2-block, 4 y-blocks, 3 1's
5. 1 y^2-block, 4 y-blocks, 4 1's
6. 1 x^2-block, 1 y^2-block, 2 xy-blocks
7. 2 y^2-blocks, 5 y-blocks, 2 1's
8. 2 x^2-blocks, 1 y^2-blocks, 3 xy-block
9. 2 x^2-blocks, 2 x-blocks, 2 xy-blocks
10. 1 x^2-block, 1 y^2-block, 2 xy-blocks, 3 x-blocks, 3 y-blocks, 2 1's

11. Take one x^2-block and 6 x-blocks. Using all seven of these blocks each time, and any number of 1's, form as many different rectangles as you can. Write down the dimensions of each rectangle formed.

Completing the Square

Materials needed: Set of commercial tiles or hand-made tiles such as those described in the Factoring Trinomials with Tiles lab.

Directions: In this worksheet you are given some blocks and you have to decide how many yellow unit blocks you need to add to *make a square.*

1. (a) Take an x^2-block and 2 x-blocks. Show how you can make a square with these blocks if you add one yellow unit block.
   (b) What are the length and width of the square you built? length _________ width ________
   (c) Complete the following. The blank on the left side represents what you have to add. The blank on the right represents the length of the side of the square that you made.

$$x^2 + 2x + \_\_ = x^2 + 2x + (\_\_)^2 = (\_\_\_\_\_)^2$$

2. (a) Take an x^2-block and 4 x-blocks. How many yellow unit blocks must you add to make a square? _______
   (b) What are the length and width of the square you built? length _________ width ________
   (c) Complete the following: $x^2 + 4x +$ ___ $=$ $x^2 + 4x + ($ ___$)^2$ $= ($ _________ $)^2$

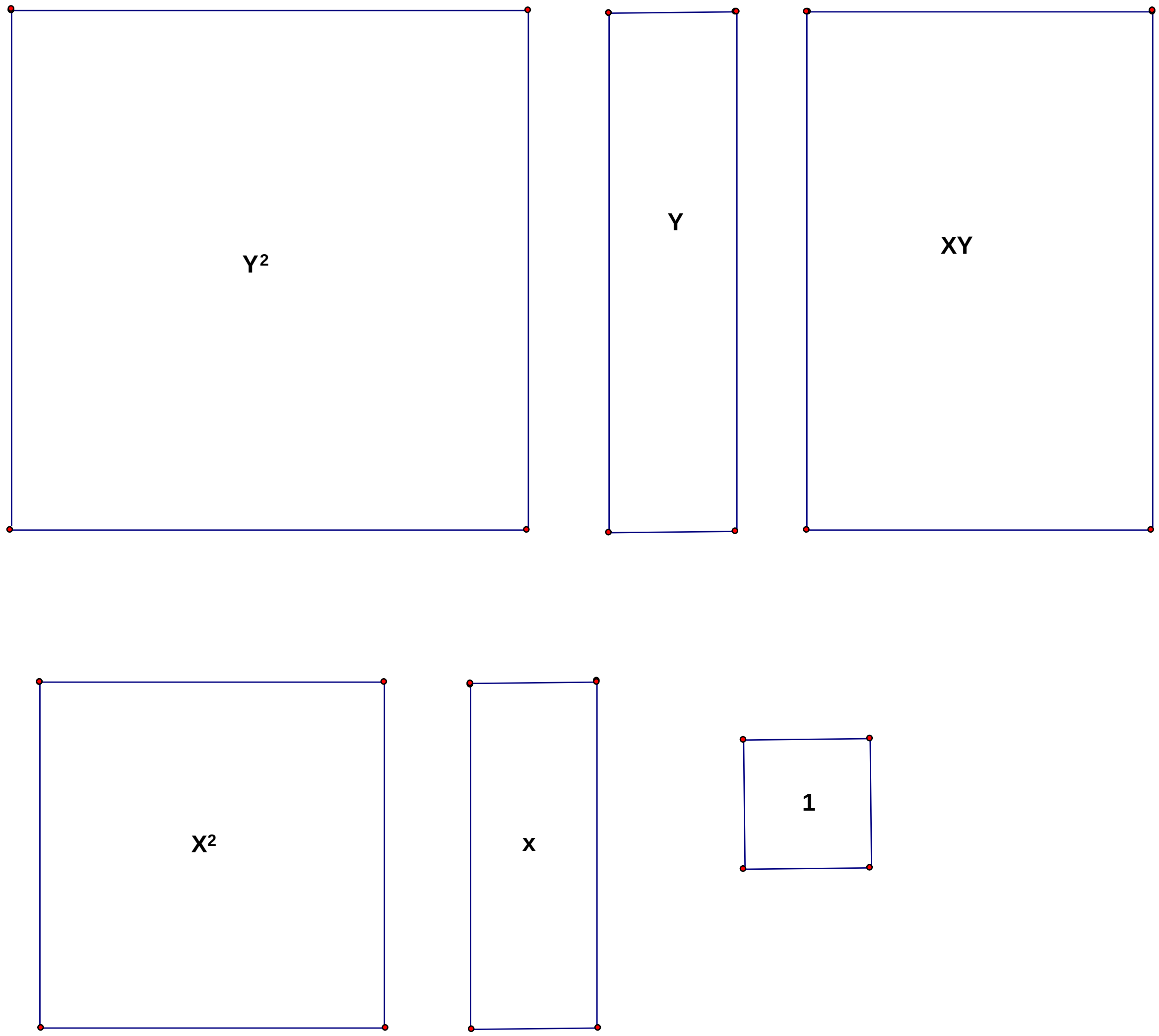
3. (a) Take an x^2-block and 6 x-blocks. How many yellow unit blocks must you add to make a square? ___________
   (b) What are the length and width of the square you built? length _________ width ________
   (c) Complete the following: $x^2 + 6x +$ ___ $= x^2 + 6x + ($ ___$)^2$ $= ($ _________ $)^2$

4 (a) Suppose you take one x^2-block and 100 x-blocks. Can you predict how many unit blocks you would have to add to make a square? ______ How many? ___________
   (b) What would the length and width of your square be? length _________ width ________
   (c) Complete the following: $x^2 + 100x +$ ___ $= x^2 + 100x + ($ ___$)^2$ $= ($ _________ $)^2$

5. (a) Suppose you take one x^2-block and *one* x-block. Imagine you could split the x -block into two blocks, each ½ by x. How much of a yellow block would you have to add to complete a square? Draw a diagram of this situation. Make sure you label all the dimensions.
   (b) What would the length and width of your square be?
   length _________ width ________
   (c)Complete the following:
   $x^2 + 1x +$ ___ $= x^2 + 1x + ($ ___$)^2$ $= ($ _________ $)^2$

6. (a) Suppose you take one x^2-block and p x-blocks. Can you predict how many unit blocks you would have to add to complete the square?
   (b) What are the length and width of the square that you built? length _______ width______
   (c) Complete the following: $x^2 + px +$ ___ $= x^2 + px + ($ ___$)^2$ $= ($ _________ $)^2$

Template for factoring tile

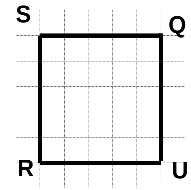


Geoboard Pythagorean Theorem

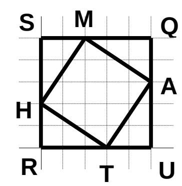
Materials needed: one 11-pin geoboard per participant. Alternately, four 7-pin boards can be placed together. Rubber bands for each participants.

Directions

1. Make this square on your geoboard with a single elastic band. Be sure to count the pegs exactly. If the distance between pegs is one unit, what is the area of square SQUR?



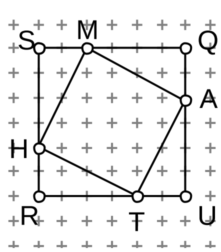
2. Add one more elastic band to make this figure. Remembering that the area of a triangle is one half the length of the base times the height, what is the area of the right triangle HRT?



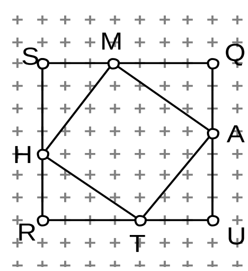
3. By subtracting the area of four triangles like HRT from the area of square SQUR, find the area of square MATH.

4. Repeat steps 1-3 with each of the figures below.

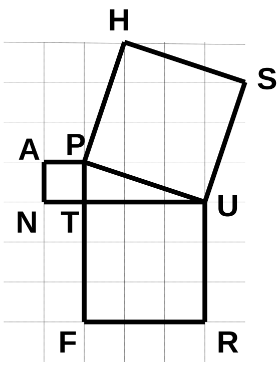
(a)



(b)

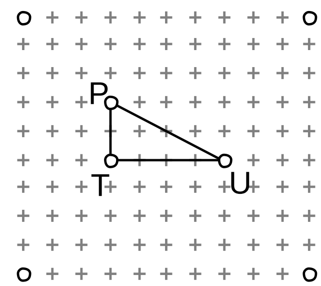


5. In this diagram a triangle PUT has squares PANT, PUSH and TURF built outwards from its sides. Make this diagram on your geoboard **by first forming the triangle PUT** and then adding the three squares. Find the areas of all three squares and put the numbers in the first row of the table below. (You will need the technique developed above to find the area of square PUSH.)



PANT	TURF	PUSH

6. On your geoboard, make a triangle similar to the one at the right. On each side of the triangle build squares PANT, TURF and PUSH, similar to the diagram in problem 5. Find the areas of all three squares and put the numbers in the 2nd row of the table above.



7. Repeat the process described in problem 6 with a triangle PUT in which PT is 2 units and TU is 5 units. You may have trouble getting all the squares on the geoboard. Put TU near the bottom and assume that you could make the appropriate square if the geoboard continued down. Be sure to allow space to enclose the square built on the side PU. Again, record your results in the 3rd row of the table above.

8. Looking at your table, describe the relationship you see between the numbers. ____________

____

9. Complete the following: If you draw squares on the sides of a right triangle, then the area of the square built on the longest side is____

____

10. To show that your generalization in problem 9 will always be true, in the space below, draw a right triangle with legs of length *a* and *b*. Draw three squares, one on each of the three sides of the triangle as we have done in the previous problems. Label the triangle PUT and the squares PANT, PUSH and TURF. Find the areas PANT and TURF. Now surround square PUSH and see if you can find its area by finding the area of the large square and subtracting the four triangles. Do your results agree with your generalization? Show your work below.

11. Your board does not have enough squares to make a triangle with sides of 8 and 15. However, by using your generalization, can you find the length of the side of the square that would be built on the longest side? Do so. Incidentally, the longest side of a right triangle is called the *hypotenuse* of the right triangle.

12. Draw a right triangle with legs of length 6 units and 8 units. Find the length of the hypotenuse. Then draw semicircles on each of the three sides, using the side as the *diameter* of the semicircle. Find the areas of the three semicircles in terms of π. Do you see a relationship between the three areas?

Pythagorean Triples

1. A triangle that appears many times in questions about Pythagoras' Theorem is the 3-4-5 triangle. You will notice that the three sides are consecutive integers. Is there another right triangle whose sides are consecutive integers? To answer this question, call the length of the shortest side n. What is the length of the next longest side? __________ and the length of the longest side? __________ Write an equation that states that the sum of the squares of the two shorter sides is the square of the length of the longest. Solve this equation. How many right triangles have sides whose lengths are consecutive integers? _________

2. Another feature of the 3-4-5 triangle is that the hypotenuse is one unit longer than the longer of the other two sides. Is there another right triangle with this property? To answer this question you will need two variables. Label the length of the shortest side of the right triangle, a. Call the hypotenuse c. What is the length of the remaining side, knowing it is one unit shorter than the hypotenuse? _________ Write an equation that states that these three numbers are the sides of a right triangle. Solve the equation for c in terms of a.

3. If the shortest side a of the right triangle in Question 2 is 5 units long, use the equation you found in Question 2 to find the lengths of the other two sides.

4. Repeat question 3, using 7, 9 and 11 for the lengths of the shortest side. Enter the data you have collected from Questions 2 and 3 in the table below.

Right Triangles With Hypotenuse One Unit Longer than its next longest side.

short side	next longest side	hypotenuse
3	4	5
5		
7		
9		
11		

5. Add the hypotenuse to the next longest side and place that sum in a fourth column in the table in question 4. Compare the results in column 4 with those in column 1. State in words any relationships that you see. Now verify algebraically that this relationship will always hold.

6. Notice that all the numbers in the middle column of the table are divisible by 4. Divide each of the numbers by 4 and place them in a column 5. Do you see a pattern in the numbers in this column?

7. Is it possible to have a Pythagorean Triple in which one side that is *two* units less that the hypotenuse? Modify the approach in step 2 to determine a relationship between the hypotenuse, c, and the shorter side, a. Generate a table similar to the one in step 4. What are some patterns that you see?

8. Each of these Pythagorean triples found in question 4 contains exactly one even number and two odd numbers. Is it possible for all three of the numbers in a triple to be even? Explain.

9. Why it is impossible for all three to be odd numbers? Can exactly two of the numbers in a Pythagorean triple be even numbers? Give reasons for your answers.