EXETER MATH INSTITUTE

HANDS-ON ALGEBRA I
PART A

Hands-on Algebra I – Part A

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Five Easy Pieces

Materials needed: an 8 ½ x11 piece of paper and pair of scissors for each participant.

Directions:

1. Begin with a blank piece of paper. Take the upper left-hand corner, fold it across to the right edge, and down towards the bottom in such a way as to form the largest possible right triangle. Cut off the flap at the bottom and unfold the paper so that you have the largest square possible.

2. Fold the square in half to form a rectangle.

3. Without unfolding, fold the rectangle in half to form a square. Continue by folding the square in half to form another rectangle. Fold the rectangle in half to form a small square.

4. Unfold the paper back to the original square. Cut out 1/4 of the original square and label it "U". Set it aside.

5. Cut another 1/4 of the original square. Cut out 4 smaller squares and label each "E". Set these aside.

6. Cut another 1/4 of the original square. Cut across the diagonal to form two congruent triangles and label each triangle "O". Set these aside.

7. On the last 1/4 of the original square place a dot at the midpoint of each side (on the fold line). Cut along the diagonals from dot to dot to form 4 congruent triangles. Label each small triangle "A". Set these aside.

8. Label the remaining square "I".

9. Complete the following chart by comparing the pieces you have. The first is done for you.

Question	Numerical answer	Written expression
How many As does it take to cover an E?	2	2A = E
How many As does it take to cover an I?		
How many As does it take to cover an O?		
How many As does it take to cover an U?		
How many Es does it take to cover an U?		
How many Es does it take to cover an O?		
How many Es does it take to cover an I?		

10. Referring to your written expressions,

(a) If 4A = I and 8A = U, show algebraically the number of Is needed to equal one U.

(b) If 4A = I and I = 2E, show algebraically the number of As needed to equal one E.

11. If you could cut up piece I, would it cover piece O? Justify your answer without actually cutting the pieces.

12. Write down all the possible ways that you can make the square U.

13. Take one E, one I and one U. These three squares renamed themselves: Bill, Jill and Lill.
Bill said, “I am bigger than Lill.”
Jill said, “I am bigger than Bill.”
Who are we?

14Take one A, one E and one O. These three pieces renamed themselves Ali, Bet and Tim.
Ali said, “I am twice as large as Bet.”
Bet said, “I am the same shape as Tim.”
Who are we?

15. Take one A, one E, one O and one U.
These four pieces renamed themselves: Mary Larry, Harry and Cary.
Mary said, “I am a fourth as large as Harry.”
Larry said, “I am bigger than Harry.”
Who are we?

16. Take one A, one I, one O and one U. These four pieces re-named themselves Floe, Joe, Moe and Woe.
Joe said, “I am twice as large as Moe.”
Floe said, “I am the same shape as Woe.”
Woe said, “I am smaller than Moe.”
Who are we?

Guess and Check Part A

Solving word problems often takes more time and effort that a computational math problem. However, they often can be done by thinking about how you might check an answer if you finally found one. In solving the following problems, try to suspend any previous methods that you might have learned and follow the directions step-by-step.

1. The length of a certain rectangle exceeds its width by exactly 8 cm. The perimeter of the rectangle is 66 cm. What are its dimensions?

   Although you may be able to solve this problem using a method of your own, try the following approach, which begins by guessing the width of the rectangle. Study the first row of the table below, which begins with a guess of 10-cm. for the width. Now make your own guess for the width and use it to fill in the next row of the table. Be sure to show the arithmetic in detail that you used to complete each entry. If this second guess was not correct, try again.

width	length	perimeter	desired perimeter	check?
10 cm.	10 + 8 = 18 cm.	2(10)+2(18) = 56	66	no

Even if you have guessed the answer, substitute a *w* in the first column and fill in the length and perimeter entries in terms of *w*.

Finally, set your expression for the perimeter equal to the desired perimeter. Solve this resulting equation and thus solve the given problem.

This approach to creating equations and solving problems is called the ***guess-and-check*** method.

2. A portable CD player is on sale for 25% off its original price. The sale price is $30. What was the original price?

In the table below, one guess has been made, which was not the correct answer. Make a guess of your own and put it in column 1. Continue to fill in across the row with the appropriate values. If your guess does not yield the correct solution, use another row of the table and guess again. Be sure to show your calculations in the table.

Finally, place a variable in the first column and continue across the row, performing the same operations with the variable as you did with numbers, as far as possible.

Form an equation by setting your expression in the "sale price" column equal to the number in the "given sale price" column. Solve the resulting equation in the space below the table and thus solve the given problem.

Original price(**guess**)	25% of original price	sale price	given sale price	equal?
60	.25(60)=15	60-15=45	30	No

3. A group of students are in a room. After 12 students leave, 3/5 of the original group remain. How many were in the room originally?

Once again, begin by making a guess at the correct answer. Eventually, put a variable in the "guess" column, complete each column as in the previous examples and finally form an equation that can help you solve the problem. Remember to show your work, both in the table and in solving the equation.

Original number of students(**guess**)	number who left	number remaining	3/5 of the original	check?

4. There are 1800 students in a high school. The ratio of male to female students is 4:5. How many boys are there?

As in previous examples, begin by making some guesses and then use a variable to form an equation. Solve the equation in the space below the chart.

number of boys(**guess**)	total number in school	number of girls	ratio of boys to girls	is ratio 4/5?

5. At noon, you start out walking to a friend's house at 4 mph. At the same time, your friend starts biking towards you at 12 mph. If your friend's house is 8 miles away, how much time will elapse before you two meet?

In the table below, put your own headings on the columns before making a guess. It is usually a good idea to label the first column with what you are trying to find. After putting the titles on the columns, make your guess, form your equation and solve it as in the previous problems. You may use fewer or more columns than are shown, if you wish.

6. Kim just bought ten gallons of gasoline, the amount of fuel used for the last 360 miles of driving. Being a curious sort, Kim wondered how much fuel had been used in city driving (which takes one gallon for every 25 miles) and how much had been used in freeway driving (which takes one gallon for every 40 miles). Determine the number of gallons used on each part of the trip.

   This time, try to determine what headings you might put at the top of each column. Start by thinking about what you wish to find and use that as the title of the first column. Thinking about what else you can determine should help you with the headings of the other columns. There are several possibilities.

   Finally, form and equation and solve it.

7. A train is leaving in 12 minutes and you are 1 mile from the station. Assuming you can walk at 4 miles per hour and run at 8 miles per hour, how much time can you afford to walk before you must begin to run in order to catch the train? Use the *guess-and-check* method to solve this problem.

Rings

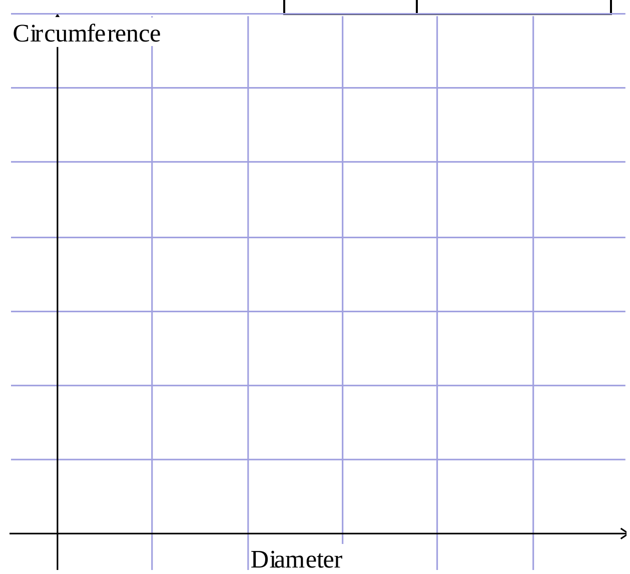
Materials needed: For each group, set of 5 rings of varying diameters (plastic lids, covers etc.), string to measure circumferences (alternately, you may roll the disks along the meter stick to measure the circumference), meter stick.

Directions:

1. For each of five different size lids, measure the diameter of the lid in centimeters to the nearest tenth. Then measure the circumference of the lid also to the nearest tenth of a centimeter. Record your data in the table at the right.

diameter	circumference

2. Plot your data on the grid at the right. Use the diameters as the independent variable.



3. Now graph the data using your calculator. Is your calculator graph similar to your graph in Step 2? What pattern does the data seem to follow?

___________________________ _____________________.

4. Select <u>two</u> points from your data that seem to be representative of the pattern and record them here.

first point ____________ and second point _______________.

5. Find the equation of the line through these two points, showing your work here.

Write your equation in slope-intercept form: y = ___________________________

6. Graph your equation along with your data. Explain what the number you computed for the slope means in terms of the measurements you made.___________________________________

____

Is the number close to what you might have predicted? _____ Explain. ___________________

____

7. What is your y-intercept? ________ If your work was very accurate, what would you expect the y-intercept be? ________ Explain. ____

8. If you had a lid that was 250 centimeters in diameter, what would be its circumference according to your equation?

9. According to your equation, what should be the diameter of a lid that has a circumference of 400 centimeters?

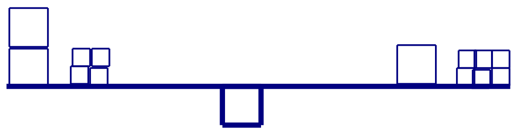
10. Bert computed the equation for his data to be $y = 3.13x$. Ernie found his equation to be $y = 3.13x + 0.02$. Each would like to use their equation to find the circumference of a lid whose diameter was found to be 2.5 feet. Explain why Ernie must convert the 2.5 feet to cm while Bert does not.

Balance Beam Lab

Materials needed: For each participant or pair of participants about 8 large blocks (dice in this lab) and about 15 smaller blocks (small wooden blocks in this lab) are needed. Candy such as caramel candy cubes (large blocks) and Starbursts (small blocks) can also be used. -

Directions:

1. Take a ruler and using two dice (one is behind the one shown in the picture below) as a fulcrum, balance the ruler as well as you can. On one side, place 2 large dice and 4 wooden blocks. On the other end, place 1 large die and 6 wooden blocks. Try to keep them towards the end with the dice on the outside. One arrangement is pictured below.

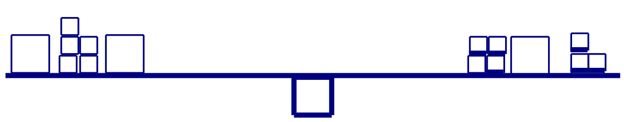


   Now try to remove blocks and dice so that you can determine how many blocks balance one die. Do this in such a way that the ruler is balanced at all times. (For example, you can take one block from the left side as long as you remove one block from the right side at the same time.)

   Record your result. _______ blocks equal one die.

2. Using a B for block and D for dice, write an equation that models the original grouping of blocks and dice as pictured above. Then show symbolically all the steps you took to determine your answer.

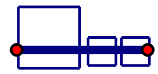
3. Clear your balance (ruler) and then form the situation pictured below using the large dice and blocks. Be sure that it will balance. Then try to remove blocks and dice, keeping the ruler balanced at all times. Your result should match the result obtained in problem 1.

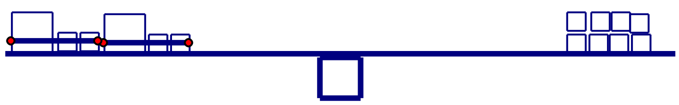


4. Using a B for block and D for dice, write an equation that models the original grouping of blocks and dice as pictured above. Then show symbolically all the steps you took to determine your answer.

5. Clear your balance (ruler) once again. On one side of the balance place 5 dice and 3 blocks and on the other side place 3 dice and 7 blocks. Try to stack the blocks and dice, and place them towards the ends of the ruler. Be sure that the ruler is balanced. Then try to remove dice and blocks as before. This time, you should reach the situation where 2 dice is balancing 4 blocks. Decide how to physically determine that 1die balances 2 blocks and also decide mathematically why you can do what you did.

6. Clear your balance. Take two blocks and one die and put a rubber band around them as pictured at the right. Do this again once more, making a total of two groupings. Place these on the left side of your ruler , stacking one on top of the other. Balance them by placing 8 blocks on the right side of the ruler as pictured.





The challenge now is to remove blocks and dice, as you did previously, to find the number of blocks that equal one die. However, before the blocks on the LHS can be removed from the ruler, what must be done first?

____

Complete that step and then proceed as before.

7. Problem number 6 can be modeled in symbols as follows: $\mathbf{2(\boldsymbol{D} + 2\boldsymbol{B}) = 8\boldsymbol{B}}$

If you wish to now show symbolically the steps taken in problem 6, what algebraic operation must be performed to remove the parentheses?

____

What physical step does that parallel in your work in problem 6?

____

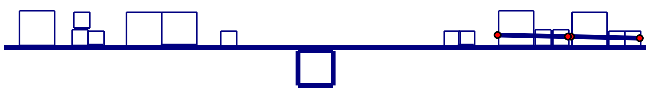
Perform this step and then continue to show the number of blocks that balance one die.

8. Returning to the set-up in problem 6, you may notice that there is a second way to approach the problem. What might be another first step?

____

Show this step symbolically by starting with **2(*D* + 2*B*) = 8*B***

9. Construct the situation pictured below on your balance and then proceed to remove blocks and dice to again show the number of blocks that balance one die.



10. Model the problem with variables and show symbolically how you arrived at your answer.

Factoring Trinomials With Tiles

Materials needed: Set of commercial tiles or hand-made tiles such as those on the next page.

Directions: Count out the following number of tiles for each group/pair of participants.

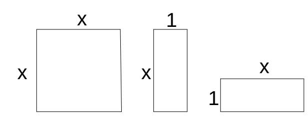
	Number needed
- small blue squares x by x called x^2-blocks	3
- large blue squares y by y called y^2-blocks	2
- small yellow 1 by 1 squares called 1's	10
- blue 1 by x rectangles called x-blocks	6
- blue 1 by y rectangles called y-blocks	5
- blue x by y rectangles called xy-blocks	3

Make a rectangle

With the above resources you are going to try to make rectangles. For each set go through the following steps.

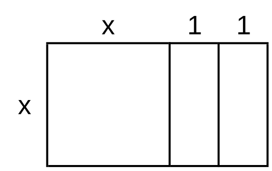
1. Write an expression for the sum of the areas of the resources. Combine like terms whenever possible.
2. Fit all the pieces together to form a rectangle.
3. Write the length and width of your new rectangle.
4. Write an expression for the area of your rectangle in the form (*length*)(*width*).
5. Write an equation that states that the sum of the areas of the individual resources (the expression you wrote in 1.) equals the area of your new rectangle (the expression you wrote in 4.)

example



Given the resources 1 x^2-block, 2 x-blocks:

1. The sum of the areas is $x^2 + x + x = x^2 + 2x$
2. Fit the pieces to form a rectangle (see lower diagram)
3. The length is $x + 2$, the width is x
4. An expression for the area is $x(x + 2)$
5. $x^2 + 2x = x(x + 2)$



you try these

1. 1 x^2-block, 3 x-blocks, 2 1's
2. 1 x^2-block, 2 x-blocks, 1 1 (yellow square)
3. 2 x^2-blocks, 4 x-blocks
4. 1 y^2-block, 4 y-blocks, 3 1's
5. 1 y^2-block, 4 y-blocks, 4 1's
6. 1 x^2-block, 1 y^2-block, 2 xy-blocks
7. 2 y^2-blocks, 5 y-blocks, 2 1's
8. 2 x^2-blocks, 1 y^2-blocks, 3 xy-block
9. 2 x^2-blocks, 2 x-blocks, 2 xy-blocks
10. 1 x^2-block, 1 y^2-block, 2 xy-blocks, 3 x- blocks, 3 y-blocks, 2 1's

11. Take one x^2-block and 6 x-blocks. Using all seven of these blocks each time, and any number of 1's, form as many different rectangles as you can. Write down the dimensions of each rectangle formed.

Completing the Square

Materials needed: Set of commercial tiles or hand-made tiles such as those described in the Factoring Trinomials with Tiles lab.

Directions: In this worksheet you are given some blocks and you have to decide how many yellow unit blocks you need to add to *make a square.*

1. (a) Take an x^2-block and 2 x-blocks. Show how you can make a square with these blocks if you add one yellow unit block.
   (b) What are the length and width of the square you built? length _________ width ________
   (c) Complete the following. The blank on the left side represents what you have to add. The blank on the right represents the length of the side of the square that you made.

$$x^2 + 2x + \_\_ = x^2 + 2x + (\_\_)^2 = (\_\_\_\_\_)^2$$

2. (a) Take an x^2-block and 4 x-blocks. How many yellow unit blocks must you add to make a square? _______
   (b) What are the length and width of the square you built? length _________ width ________
   (c) Complete the following: $x^2 + 4x +$ ___ $= x^2 + 4x + ($ ___$)^2 = ($ ________ $)^2$

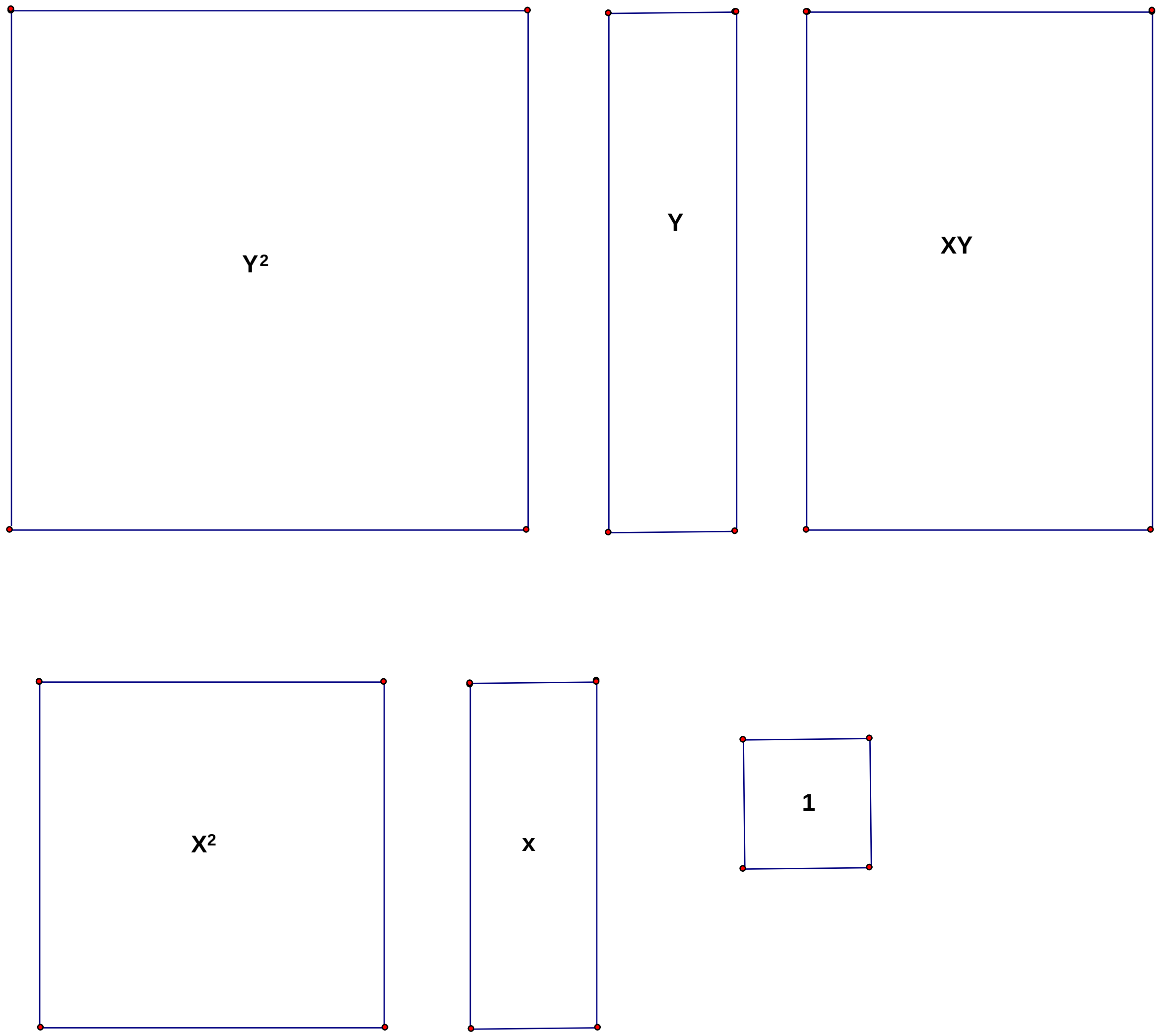
3. (a) Take an x^2-block and 6 x-blocks. How many yellow unit blocks must you add to make a square? ___________
   (b) What are the length and width of the square you built? length _________ width ________
   (c) Complete the following: $x^2 + 6x +$ ___ $= x^2 + 6x + ($ ___$)^2 = ($ ________ $)^2$

4 (a) Suppose you take one x^2-block and 100 x-blocks. Can you predict how many unit blocks you would have to add to make a square? ______ How many? ___________
   (b) What would the length and width of your square be? length _________ width ________
   (c) Complete the following: $x^2 + 100x +$ ___ $= x^2 + 100x + ($ ___$)^2 = ($ ________ $)^2$

5. (a) Suppose you take one x^2-block and *one* x-block. Imagine you could split the x -block into two blocks, each ½ by x. How much of a yellow block would you have to add to complete a square? Draw a diagram of this situation. Make sure you label all the dimensions.
   (b) What would the length and width of your square be? length _________ width ________
   (c)Complete the following: $x^2 + 1x +$ ___ $= x^2 + 1x + ($ ___$)^2 = ($ ________ $)^2$

6. (a) Suppose you take one x^2-block and p x-blocks. Can you predict how many unit blocks you would have to add to complete the square?
   (b) What are the length and width of the square that you built? length _______ width______
   (c) Complete the following: $x^2 + px +$ ___ $= x^2 + px + ($ ___$)^2 = ($ ________ $)^2$

Template for factoring tile



Springs

Materials needed: For each pair or group, one spring(½ of a Slinky works), a bucket, 2 meter sticks, M&Ms or substitute to fill the buckets.

Directions:

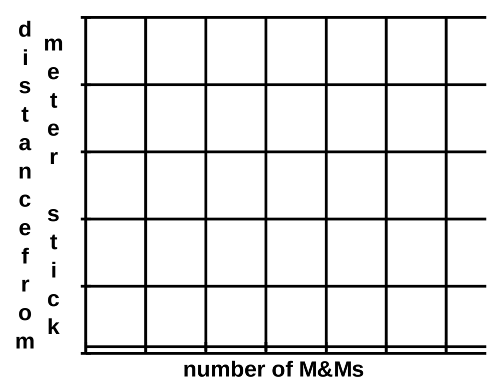
1. Place one meter stick between two chairs and slip one end of the slinky over it so that it hangs down. Hang the bucket from the other end of the slinky so that the bottom of the container is parallel to the floor.

2. Measure the distance, in centimeters**, from the bottom of the empty cup up to the meter stick** that is holding the slinky. *Record your answer* here ____________.

3. For trial number 1, place 4 M&Ms in the container and measure the distance from the meter stick to the bottom of the bucket. Record your results in the table at the right. Continue to place more M&M candies in the bucket, four at a time. Each time, measure and record the distance from the meter stick to the bottom of the canister.

trial number	total number of M&Ms	distance from meter stick
1		
2		
3		
4		
5		
6		

4. Using your calculator, make a scatterplot of the data found in step 3. In your graph, plot the total number of M&Ms on the horizontal axis and the distance from the meter stick plotted on the vertical axis. Make a copy of your graph on the graph provided on this paper. Label the points with their coordinates.



5. Your data should look like it falls on a straight line. Select the two points from your data that seem to most represent the data and write their coordinates here.

   (first point) ____________ (second point) _______________.

   Find the equation of the line through these two points, showing your work here.

   Write your final equation here: y = ___________________________

6. Graph your line along with your scatterplot. Does your line approximate your data?

7. The slope of your line is a “rate”. Explain the meaning of the specific number you computed for your slope in terms of M&Ms and the distances measured.

8. What is the significance of the y-intercept of your line? How is it related to the measurement made in step 2 of this lab?

9. Using your equation, predict the distance that the bucket would be from the meter stick if 36 M&Ms were placed in the bucket?

10. Using your equation, find **the least number** of M&Ms that would be needed to stretch the bucket and spring to a total length of at least 100 cm?

11. Create a third column of data by subtracting your initial length of the spring and bucket without any candy, from each length in your original data. Plot this new data on the vertical axis while leaving the horizontal axis as the total number of M&Ms. Pick the two points whose x-values are the same as the points chosen in question 5 and again form an equation of a line that seems to fit the data the best.

Write the equation of the line ______________________________

12. Graph your equation. What should the y-intercept be? ___________________

13. Compare the graph of this line with the line graphed in question 6. How are they alike and why is this so?

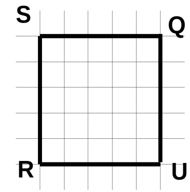
14. Explain the significance of the slope in this equation relative to the spring and the M&Ms.

Geoboard Investigation

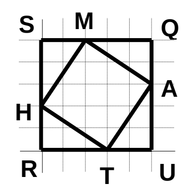
Materials needed: one 11-pin geoboard per participant. Alternately, four 7-pin boards can be placed together. Rubber bands for each participants.

Directions

1. Make this square on your geoboard with a single elastic band. Be sure to count the pegs exactly. If the distance between pegs is one unit, what is the area of square SQUR?



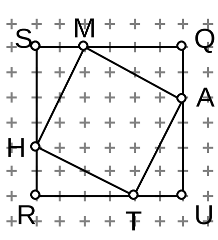
2. Add one more elastic band to make this figure. Remembering that the area of a triangle is one half the length of the base times the height, what is the area of the right triangle HRT?



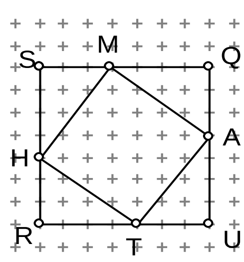
3. By subtracting the area of four triangles like HRT from the area of square SQUR, find the area of square MATH.

4. Repeat steps 1-3 with each of the figures below.

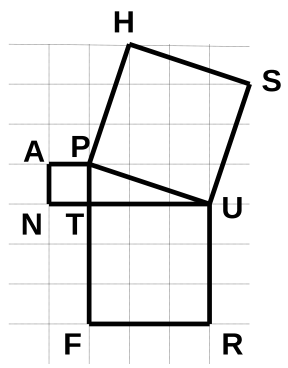
(a)



(b)

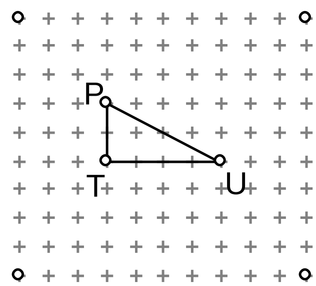


5. In this diagram a triangle PUT has squares PANT, PUSH and TURF built outwards from its sides. Make this diagram on your geoboard **by first forming the triangle PUT** and then adding the three squares. Find the areas of all three squares and put the numbers in the first row of the table below. (You will need the technique developed above to find the area of square PUSH.)



PANT	TURF	PUSH

6. On your geoboard, make a triangle similar to the one at the right. On each side of the triangle build squares PANT, TURF and PUSH, similar to the diagram in problem 5. Find the areas of all three squares and put the numbers in the 2nd row of the table above.



7. Repeat the process described in problem 6 with a triangle PUT in which PT is 2 units and TU is 5 units. You may have trouble getting all the squares on the geoboard. Put TU near the bottom and assume that you could make the appropriate square if the geoboard continued down. Be sure to allow space to enclose the square built on the side PU. Again, record your results in the 3rd row of the table above.

8. Looking at your table, describe the relationship you see between the numbers. ____________

____

9. Complete the following: If you draw squares on the sides of a right triangle, then the area of the square built on the longest side is____

____

10. To show that your generalization in problem 9 will always be true, in the space below, draw a right triangle with legs of length *a* and *b*. Draw three squares, one on each of the three sides of the triangle as we have done in the previous problems. Label the triangle PUT and the squares PANT, PUSH and TURF. Find the areas PANT and TURF. Now surround square PUSH and see if you can find its area by finding the area of the large square and subtracting the four triangles. Do your results agree with your generalization? Show your work below.

11. Your board does not have enough squares to make a triangle with sides of 8 and 15. However, by using your generalization, can you find the length of the side of the square that would be built on the longest side? Do so. Incidentally, the longest side of a right triangle is called the *hypotenuse* of the right triangle.

12. Draw a right triangle with legs of length 6 units and 8 units. Find the length of the hypotenuse. Then draw semicircles on each of the three sides, using the side as the *diameter* of the semicircle. Find the areas of the three semicircles in terms of π. Do you see a relationship between the three areas?

Pythagorean Triples

1. A triangle that appears many times in questions about Pythagoras' Theorem is the 3-4-5 triangle. You will notice that the three sides are consecutive integers. Is there another right triangle whose sides are consecutive integers? To answer this question, call the length of the shortest side *n*. What is the length of the next longest side? _________ and the length of the longest side? _________ Write an equation that states that the sum of the squares of the two shorter sides is the square of the length of the longest. Solve this equation. How many right triangles have sides whose lengths are consecutive integers? ________

2. Another feature of the 3-4-5 triangle is that the hypotenuse is one unit longer than the longer of the other two sides. Is there another right triangle with this property? To answer this question you will need two variables. Label the length of the shortest side of the right triangle, *a*. Call the hypotenuse *c*. What is the length of the remaining side, knowing it is one unit shorter than the hypotenuse? ________ Write an equation that states that these three numbers are the sides of a right triangle. Solve the equation for *c* in terms of *a*.

3. If the shortest side *a* of the right triangle in Question 2 is 5 units long, use the equation you found in Question 2 to find the lengths of the other two sides.

4. Repeat question 3, using 7, 9 and 11 for the lengths of the shortest side. Enter the data you have collected from Questions 2 and 3 in the table below.

Right Triangles With Hypotenuse One Unit Longer than its next longest side.

short side	next longest side	hypotenuse
3	4	5
5		
7		
9		
11		

5. Add the hypotenuse to the next longest side and place that sum in a fourth column in the table in question 4. Compare the results in column 4 with those in column 1. State in words any relationships that you see. Now verify algebraically that this relationship will always hold.

6. Notice that all the numbers in the middle column of the table are divisible by 4. Divide each of the numbers by 4 and place them in a column 5. Do you see a pattern in the numbers in this column?

7. Is it possible to have a Pythagorean Triple in which one side that is *two* units less that the hypotenuse? Modify the approach in step 2 to determine a relationship between the hypotenuse, c, and the shorter side, a. Generate a table similar to the one in step 4. What are some patterns that you see?

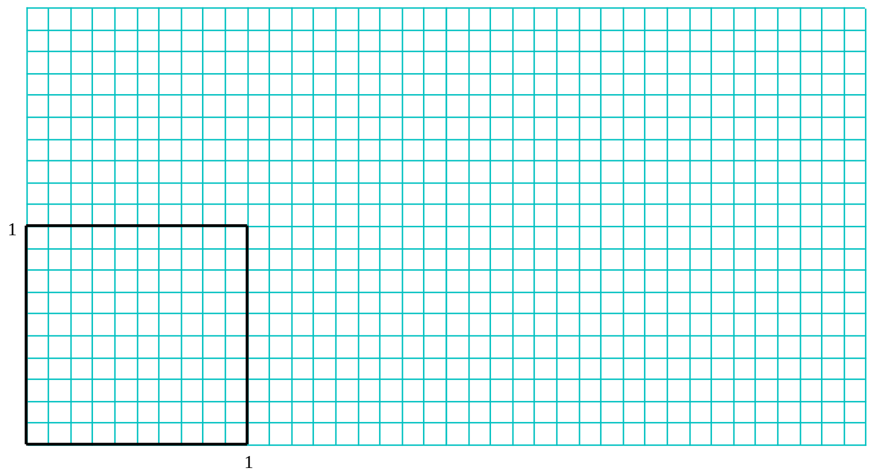
8. Each of these Pythagorean triples found in question 4 contains exactly one even number and two odd numbers. Is it possible for all three of the numbers in a triple to be even? Explain.

9. Why it is impossible for all three to be odd numbers? Can exactly two of the numbers in a Pythagorean triple be even numbers? Give reasons for your answers.

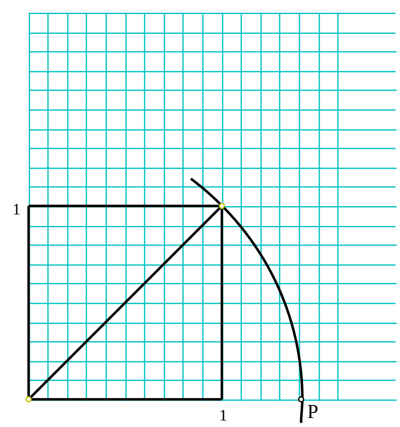
Irrational Number Worksheet

Materials needed: Graph paper, straight edge, compass

1. Using the straight edge, construct a square of unit length in the first quadrant using the points (0,0), (1,0), (0,1), and (1,1) as the vertices. In order for the square to be a good size, use 10 blocks on the graph paper to equal one unit length. Put the origin in the lower left hand corner of the graph paper with the longest side of the graph paper arranged horizontally.



2. Use the straight edge to construct the diagonal from (0,0) to (1,1). Use the Pythagorean Theorem to confirm the exact length of this diagonal is $\sqrt{2}$.

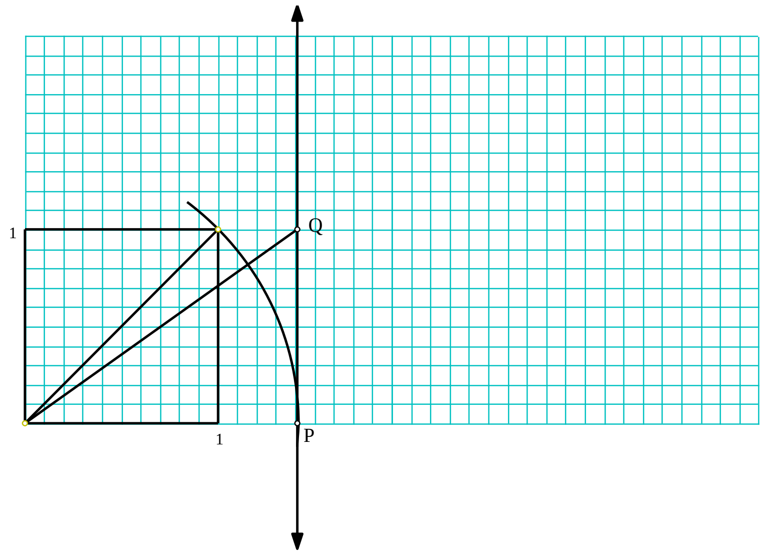


3. By placing the point of the compass at (0,0) and the pencil end of the compass on (1,1), construct an arc of a circle that intersects the *x*-axis at a point labeled P. What is the exact distance from the origin to point P?

4. Hopefully you answered the above question with $\sqrt{2}$. Using the squares on your graph paper, give an approximation to the nearest tenth for $\sqrt{2}$. Use your calculator to check to see if your approximation is correct.

5. The $\sqrt{2}$ is an example of an *irrational number*. Irrational numbers exist on the number line, but they cannot be expressed exactly as a decimal or a fraction. Even the number on your calculator is only a very accurate approximation for $\sqrt{2}$. *Rational numbers* are those numbers on the number line that can be expressed as decimals or fractions. Use the calculator approximation to obtain a decimal approximation for $\sqrt{2}$ .Between what two integers on the *x*-axis would you locate $\sqrt{2}$? If you divide your *x*-axis into tenths, between what two tenths would you locate $\sqrt{2}$? Now determine two rational numbers that are 0.001 apart, yet the $\sqrt{2}$ lies in between them. How about two rational numbers that are 0.00001 apart that have $\sqrt{2}$ in between them?

6. Using the straight edge, construct a vertical line through point P. Label where this vertical line intersects the horizontal line 1 unit up as point Q. Use the right triangle formed by the origin, and points P and Q to determine the exact length of the segment from the origin to point Q.



7. Hopefully, you calculated this length to be exactly $\sqrt{3}$. Now use the compass in a manner similar to what you did in the previous example to determine the location of $\sqrt{3}$ on the *x*-axis. Counting blocks on your graph paper, give an approximation for $\sqrt{3}$, and then check your answer using the calculator.

8. Use the calculator approximation of $\sqrt{3}$ to find two rational numbers on the *x*-axis line 0.01 apart, yet the $\sqrt{3}$ lies in between them. How about two rational numbers that are 0.000001 apart that have $\sqrt{3}$ in between them?

9. Use the straight edge and compass method along with the Pythagorean Theorem to locate $\sqrt{5}$ on the *x*-axis the same way you located $\sqrt{2}$ and $\sqrt{3}$.

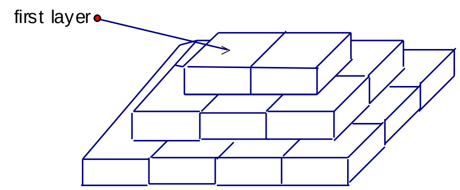
A challenge: Locate $\sqrt{7}$ on the *x*-axis through a compass- straight edge construction.

Stacking Starbursts

Materials needed: For each group, 20 starbursts or similar objects.

Directions:

1. Each group of people should have about 20 Starbursts. Each person in the group should have his/her own worksheet and should complete it as the group works through the following lab.



2. Begin by building the 3-layer figure at the right with Starbursts. Note that each layer is rectangular in shape and each Starburst covers the intersection of four Starbursts on the row below. Assume that layer #1 is the top layer of 2 Starbursts.

3. Record the number of Starbursts used in each layer in the table below.

4. You do not have enough Starbursts to actually construct a fourth layer under the three already there. However, using those in the top three layers can help you determine how many you would need for this four layer. Record that number in the table.

5. Do you see a pattern forming? Try to use this pattern to complete the table for layers 5 and 6

6. Is the data we have collected linear? ________ How do you know?

   ____

Layer number	Number of starbursts	Difference between 2nd column values	Difference between 3rd column values
1			
2			
3			
4			
5			
6			

7. In the third column of the table record the successive difference between the values in the second column. Are they constant? Does the data in column 3 form a linear function?

8. In the fourth column record the successive differences in the values in the third column. What do you notice? Our original data in columns 1 and 2 is therefore what degree of polynomial function?

9. Sketch a graph of your data in columns 1 and 2. It should confirm your previous answers.

10. We would now like to write a function that would give us the number of Starbursts used, *y*, given the number of the layer, *x*. We now know it would be an equation of the form $y = ax^2 + bx + c$. To begin, using the patterns in the table, how many Starbursts would there be in the 0 layer? What then should be the value of *c*?

11. Using two of the sets of data points, form two equations and solve them simultaneously for *a* and *b*. You can now write the desired equation.

Paper folding

Materials needed: 8 ½ x 11 paper, a few sheets of newspaper or larger paper to demonstrate maximum number of folds possible.

Directions:

1. How many times is it possible to fold a piece of paper? Take a piece of 8½ x 11 paper and fold it in half. You now have something that is two pages thick. Fold that piece in half again. How thick is it now? Continue folding and recording your results in the second column of the table at the right for 6 folds.

number of folds	thickness in pages	thickness in inches
1	2	0.006
2	4	
3		
4		
5		
6		

2. If one piece of paper is 0.003 inches thick, how thick would two pages be? Four pages? Record the thickness in inches in the third column of your table for all six folds.

3. It is virtually impossible possible to fold the paper 15 times. However, we could cut and stack the paper and obtain the same results. Looking back at step 1, we could have cut the paper in half and put one on top of the other. We could then have cut the pile in half and stacked it, obtaining a stack of four sheets. After doing this 13 more times for a total of 15, how many pieces of paper would be in the stack? At 0.003 inches per piece of paper, how high would the stack now be? Record these numbers in your table. Convert your answer to feet, showing your calculations.

4. Based on your work thus far, how thick in feet would you <u>estimate</u> that a stack formed by 25 cutting and stacking operations would be? Write your estimate below. Now compute the actual thickness of the stack formed by doing this operation 25 times and record your answer. Make use of your calculator to compute this answer.

   my estimate ________________ my answer from the calculator ____________________

5. Form a formula that gives the thickness *in feet* of the folded paper as a function of the number of folds and write it in the space below.

6. Mount Everest, the tallest mountain in the world is 29,028 feet high. About how many cutting and stacking operations would it take to get a stack at least this high? (Record your answer and also describe how you got it.)

7. The moon is 286,000 *miles* away. How many cutting and stacking operations would it take to form a stack that would reach the moon?

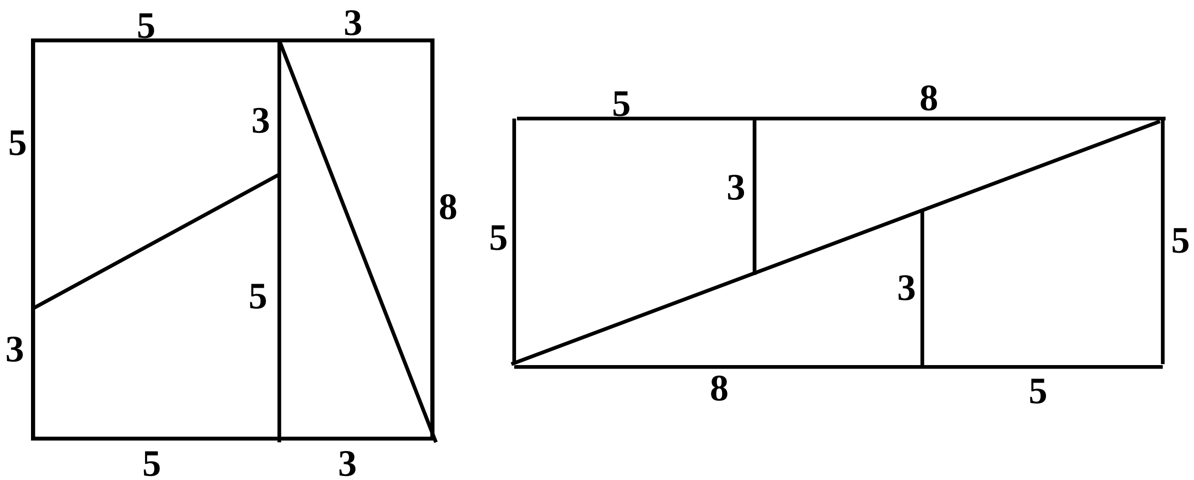
8. If we performed the cutting and stacking operation 24 times we would obtain a stack that is almost 0.8 miles high. (Verify this.)
   (a) If we would want to try to actually do this task, how many sheets of paper would be in the stack?

   (b) If each sheet in the stack was one square foot in area, what would be the dimensions of a large square piece of paper that could be used to accomplish this task?

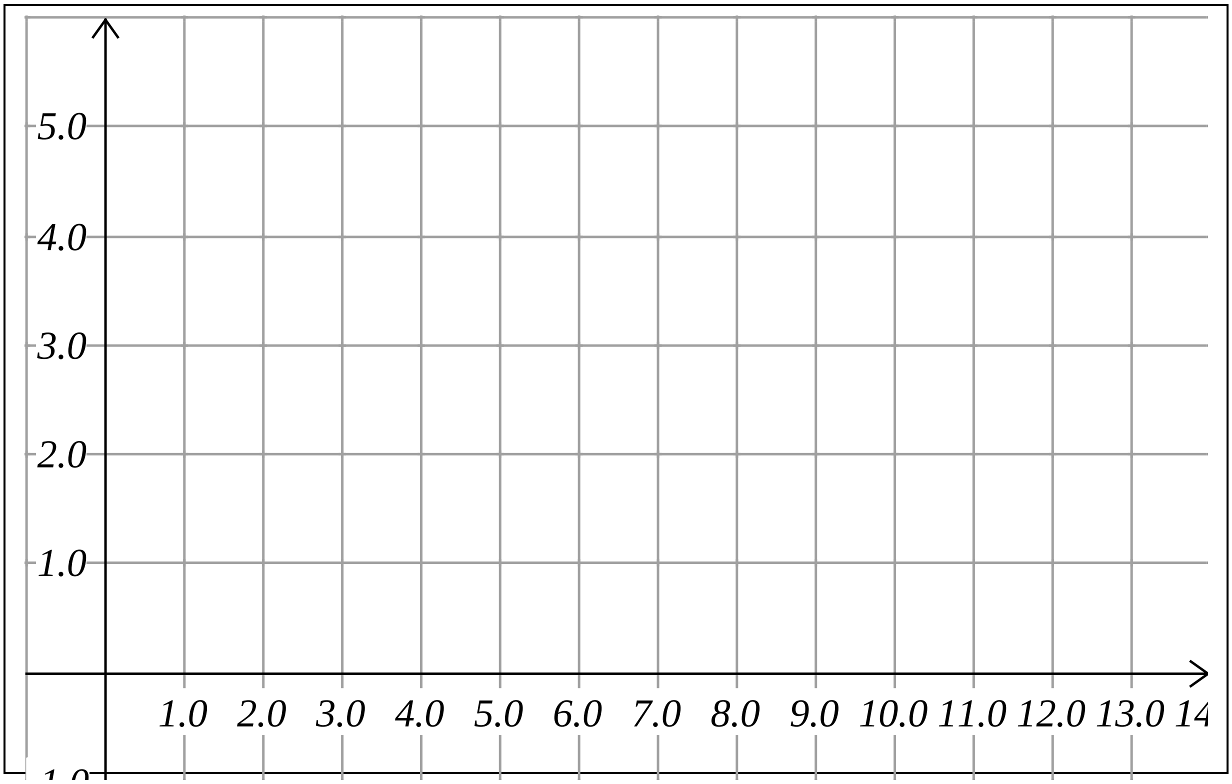
What Is Wrong With This Picture?

Directions:

Study the two pictures below. Is there anything wrong with what you see? Assume that the first figure is a square.



2. Draw the right-hand figure as accurately as possible on the graph below. Do the four pieces from the square fit perfectly in the rectangle? That is the area of the given square? ______? What is the area of the rectangle you formed? _____ Can you determine why they differ?



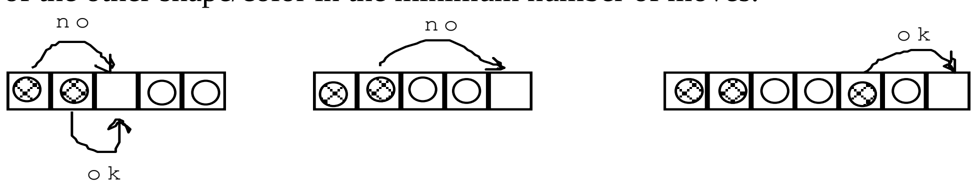
The Jumping Game

Materials needed: Two groups of different objects, four objects in each group. Colored disks work well. A sheet of paper on which to re-draw the figures in step 2 if these are not large enough to accommodate the disks.

The following game is based on these rules:

1. Moves can be made in only one direction. The objects cannot move backwards.
2. You may move into an adjacent unoccupied space.
3. You may jump a mover of the opposite shape/color, but you may only jump over one mover on each jump.

The object of each game is to completely switch the positions of the movers of one shape/color with the movers of the other shape/color in the minimum number of moves.



1. Place two different markers in the positions indicated on the game board at the right. What is the minimum number of moves necessary to completely switch their positions?

X		O

2. Now try the same thing with each of the following games and record the least number of moves needed in each game in the table provided. You may find it helpful to re-draw each game board on another piece of paper.

x	x		o	o

x	x	x		o	o	o

x	x	x	x		o	o	o	o

# of markers of one color	minimum # of moves
1	
2	
3	
4	

3. Is the pattern in the second column linear? ________ quadratic? ________ exponential? ________

Give a reason for your answer.

____

4. Determine an equation that describes the relationship between the two columns. ______________

It might be helpful to continue the pattern "backwards" to zero "# of markers of one color."

Your equation: _________________________

5. Return to the original problem and re-do the moves for each game. Record whether the move was a "slide" into the next space or a "jump" over one of the markers. For example, in the first game, the moves might be recorded as

S J S

What would be the pattern in the second game look like ? ____________________________

And the third? ____

The fourth ? ____

Fill in your values in the tale at the right: Remember that the two entries must total to be the same as the corresponding entry in your last table.

# of markers of one color	Number of jumps	Number of slides
1		
2		
3		
4		
n		

6. Let *n* represent the number of markers of one color. How could you represent the number of jumps in terms of *n*? _______

How could you represent the number of slides in terms of *n*? _______

What is the total number of moves in terms of *n*. ___________________ How does this compare with the formula obtained in step 4?

____